



Via Po, 53 – 10124 Torino (Italy)
Tel. (+39) 011 6704043 - Fax (+39) 011 6703895
URL: <http://www.de.unito.it>

WORKING PAPER SERIES

Equilibrium policy simulations with random utility models of labour supply

Ugo Colombino

Dipartimento di Economia "S. Cognetti de Martiis"

Working paper No. 15/2010



Università di Torino

Equilibrium policy simulations with random utility models of labour supply

Ugo Colombino*

University of Torino, Collegio Carlo Alberto and IZA

Abstract

Many microeconomic models of discrete labour supply include alternative-specific constants meant to account for (possibly besides other factors) the density or accessibility of particular types of jobs (e.g. part-time jobs vs. full-time jobs). The most common use of these models is the simulation of tax-transfer reforms. The simulation is usually interpreted as a comparative static exercise, i.e. the comparison of different *equilibria* induced by different policy regimes. The simulation procedure, however, typically keeps fixed the estimated alternative-specific constants. In this note we argue that this procedure is not consistent with the comparative statics interpretation. Equilibrium means that the number of people willing to work on the various job types must be equal to the number of available jobs. Since the constants reflect the number of jobs and since the number of people willing to work change as a response to the change in tax-transfer regime, it follows that the constants should also change. A structural interpretation of the alternative-specific constants leads to the development of a simulation procedure consistent with the comparative static interpretation. The procedure is illustrated with an empirical example.

JEL Classification: C35, C53, H31, J22.

Keywords: Random Utility, Discrete Choice, Labour Supply, Simulation of tax reforms, Alternative-specific constants, Equilibrium simulation.

Corresponding author:

Ugo Colombino

Department of Economics Cogneiti De Martiis, University of Torino

Via Po 53

10124 Torino

Italy

ugo.colombino@unito.it

*Financial support from the Compagnia di San Paolo and from the Italian Ministry of the University is gratefully acknowledged. Edlira Narazani (Research Assistant at the Department of Economics, University of Torino) has provided skillful programming and useful comments. I wish to thank the participants at the 1st Essex Microsimulation Workshop (Essex, September 2010) and the participants at the CHILD workshop “Basic Income Policies and Tax Reforms: Alternative Concepts and Evaluations” (Collegio Carlo Alberto, November 2009), in particular Lennart Flood for a useful discussion upon the topics addressed in this note. The usual disclaimers apply.

1. Introduction

A common practice in the specification of models of labour supply based on the discrete choice approach consists of introducing alternative-specific constants, which should account for a number of factors such as the different density or accessibility of different types of jobs, search or fixed costs and systematic utility components otherwise not accounted for. In the basic framework, the agent chooses among a set Ω of alternatives or “job” types j , non-market “jobs” (i.e. non-participation) included. In most applications, the types are defined in terms of ranges of hours of work, but more generally they might be “packages” that include hours, incomes, commuting time, degree of security etc. $U_i(j; w_i, T, \varepsilon_{ij}) = V_i(j; w_i, T) + \varepsilon_{ij}$ denotes the utility attained by agent i if a job of type j is chosen, given wage rate w_i and tax-transfer regime T , where $V_i(j; w_i, T)$ is the systematic part (containing observed variables) and ε_{ij} is a random component. In order to simplify the exposition, in this note we treat the wage rate as a characteristics of the agent, although more generally it could depend both on the agent and on the job type. By assuming that ε_{ij} is i.i.d. extreme value Type I, we get the familiar Multinomial Logit expression for the probability that a job of type j is chosen:

$$(1) \quad P_i(j; w_i, T) = \frac{\exp\{V_i(j; w_i, T)\}}{\sum_k \exp\{V_i(k; w_i, T)\}}$$

Model (1) usually does not fit the data very well. For example, if job types are defined in terms of hours of work, it tends to over-predict the number of people working below and above the modal hours. More generally, certain types of jobs might differ according to a number of systematic factors that are not accounted for by the observed variables contained in V : (a) availability or density of job-types; (b) fixed costs; (c) search costs; (d) systematic utility components. What might be called the “dummies refinement” is a simple way to account for those factors. Let us define subsets $\{S_\ell\}$ of Ω and the corresponding dummies $\{D(j \in S_\ell)\}$ such that $D(e) = 1$ iff e is true.

Clearly the definition of the subsets should reflect some hypothesis upon the differences among the job types with respect to the factors (a) – (b) mentioned above. Now we specify the choice probability as follows:

$$(2) \quad P_i(j; w_i, T) = \frac{\exp\left\{V_i(j; w_i, T) + \sum_\ell \mu_\ell D(j \in S_\ell)\right\}}{\sum_k \exp\left\{V_i(k; w_i, T) + \sum_\ell \mu_\ell D(k \in S_\ell)\right\}}$$

Many papers – although with differing focus and motivation – have adopted a similar procedure, among others: Van Soest (1995), Aaberge et al (1995, 1999, 2006, 2010), Kalb (2000), Dagsvik et al. (2006), Kornstad et al. (2007) and Colombino et al. (2010).

The main use of microeconomic models of labour supply consists of the simulation of tax-transfer reforms. Once $V(\cdot)$ and the $\{\mu_\ell\}$ are estimated, the current tax regime T is replaced by a “reform” R and a new distribution of choices can be simulated using expression (2). The results of these simulations are most commonly interpreted as comparative statics exercises. We compare two different *equilibria* induced by two different tax-transfer regimes. This interpretation is reinforced by the fact that the models are typically estimated and simulated with cross-section data. All the authors adopting the “dummies refinement” perform the simulation while leaving the $\{\mu_\ell\}$ unchanged, so that the new choice probabilities are:

$$(3) \quad P_i(j; w_i, T) = \frac{\exp\left\{V_i(j; w_i, T) + \sum_\ell \mu_\ell D(j \in S_\ell)\right\}}{\sum_k \exp\left\{V_i(k; w_i, T) + \sum_\ell \mu_\ell D(k \in S_\ell)\right\}}$$

We claim that the above procedure is not consistent with the comparative statics interpretation mentioned above. In the simplest concept of equilibrium, the number of people willing to work must equal to the number of available jobs, both in total and in the different hours ranges. Since the $\{\mu_\ell\}$ reflect – at least in part, depending on the interpretations – the number and the composition of available jobs, and since the number of people willing to work and their distribution across different job types in general change as a consequence of the reforms, it follows that in general the $\{\mu_\ell\}$ must also change. A series of papers by Aaberge et al. (1995, 1999, 2006, 2010), building on a matching model developed by Dagsvik (1994, 2000) extend the basic random utility approach to include a random choice set and provide a structural interpretation of the “dummies refinement”. We claim here that this interpretation leads very naturally to a simulation procedure that ensures the consistency with the comparative statics interpretation. For simplicity of exposition we start with considering a single individual. Letting $\delta(j)$ denote the density of available jobs of type j , under appropriate assumptions the probability that individual i is matched to a job of type j turns out to be:¹

¹ The opportunity density might be specified as individual-specific but in this illustration we assume it to be common to everyone for the sake of simplicity.

$$(4) \quad P_i(j; w_i, T) = \frac{\exp\{V_i(j; w_i, T)\} \delta(j)}{\sum_k \exp\{V_i(k; w_i, T)\} \delta(k)}.$$

The density $\delta(j)$ can be interpreted as reflecting the demand side. By adopting a convenient specification for $\delta(j)$ – as explained for example in Aaberge et al. (1999) – we end up with the following expression:

$$(5) \quad P_i(j; w_i, T) = \frac{\exp\{V_i(j; w_i, T) + \mu_0 D_0(j \in M) + \sum_{g=1}^G \mu_g D_g(j \in M_g)\}}{\sum_k \exp\{V_i(k; w_i, T) + \mu_0 D_0(k \in M) + \sum_{g=1}^G \mu_g D_g(k \in M_g)\}}$$

where M is the subset of market job-types and M_1, \dots, M_G are G subsets of M . It can then be shown that the coefficients of these dummy variables have the following interpretation:

$$(6) \quad \mu_0 = \ln\left(\frac{J}{H}\right)$$

and

$$(7) \quad \mu_g = \ln\left(\frac{J_g/J}{A_g}\right)$$

where J = number of jobs of type $j \in M$ (i.e. number of market jobs), H = number of “jobs” of type $j \notin M$ (i.e. the number of non-market “jobs”), J_g = number of jobs of type $j \in M_g$ and A_g = number of types in M_g . The presence of factors other than jobs density (such as search or fixed costs) is not incompatible with expression (6) and (7) above: indeed H and A_g can be more generally interpreted as normalizing constants that include the effect of those other factors.

2. Equilibrium conditions

For ease of exposition we start by assuming that the model contains only one dummy, $D_0(j \in M)$. It is important to distinguish the case of a finite negative elasticity of the demand for labour from the cases of perfectly elastic demand and perfectly rigid demand.

Finite negative elasticity

Let us assume that the number of available jobs depends on the average wage rate \bar{w}

$$(8) \quad J = J(\bar{w}).$$

Using (6) and (8) we can write:

$$(9) \quad \mu_0 = \mu_0(\bar{w}).$$

We then define $\pi_i(T, \bar{w}, \mu_0(\bar{w}))$ as the probability that individual i is working given tax-transfer regime T and average wage rate \bar{w} :

$$(10) \quad \pi_i(T, \bar{w}, \mu_0(\bar{w})) \equiv \frac{\exp\{V(j; \bar{w} + u_i, T) + \mu_0(\bar{w})D_0(j \in M)\}}{\sum_{j \in M} \sum_k \exp\{V(k; \bar{w} + u_i, T) + \mu_0(\bar{w})D_0(k \in M)\}}$$

where $\bar{w} + u_i = w_i$. Assuming that the observed (or simulated) choices under the current tax-transfer regime T corresponds to an equilibrium, we must have:

$$(11) \quad \sum_i \pi_i(T, \bar{w}, \mu_0(\bar{w})) = J(\bar{w}).$$

In a comparative static perspective, an analogous condition must hold under the ‘‘reform’’ R :

$$(12) \quad \sum_i \pi_i(R, \bar{w}_R, \mu_0(\bar{w}_R)) = J(\bar{w}_R)$$

where \bar{w}_R denotes the new average equilibrium wage.

Infinite elasticity

When the demand for labour is perfectly elastic, the market is always in equilibrium at the initial wage rate. However, since the number of working people in general will change under a new tax-transfer rule and since the number of jobs in equilibrium must be equal to the number of working people, it follows that the parameter $\mu_0 = \ln\left(\frac{J}{H}\right)$ must change. Rewrite expression (6) as

$$(13) \quad J = He^{\mu_0}$$

Then the equilibrium condition can be written as follows

$$(14) \quad \sum_i \pi_i(T, \bar{w}_R, \mu_{0R}) = He^{\mu_{0R}}$$

In this case \bar{w} remains fixed. Instead μ_{0R} must be directly adjusted so as to fulfill condition (14).

This case, with \bar{w} fixed and the demand absorbing any change in supply at that wage, actually corresponds to the scenario implicitly assumed in most tax-transfer simulations: however those simulations do not take condition (14) into account.

Zero elasticity

If the demand for labour is perfectly rigid (zero elasticity), the number of jobs remains fixed. The wage rate must be adjusted so that the number of people willing to work under the new regime is equal to the (fixed) number of jobs. Therefore we must have

$$(15) \quad \sum_i \pi_i(R, \bar{w}_R, \mu_0(\bar{w})) = J(\bar{w}).$$

3. Extensions

The basic framework illustrated above can be extended in many directions.

3.1. Different types of jobs

As in expression (5), we might want to account for different types of jobs. Let us consider again a single person. In this case we might for example specify $J = J(\bar{w})$ and $J_g = J_g(\bar{w})$, $g = 1, \dots, G$, which would imply the relationships $\mu_0 = \mu_0(\bar{w})$ and $\mu_g = \mu_g(\bar{w})$.²

We then define the probability that individual i is matched to a market job of type $j \in M_g$ as

$$(16) \quad \pi_i^g(T, \bar{w}, \mu_0(\bar{w}), \dots, \mu_G(\bar{w})) \equiv \sum_{j \in M_g} \frac{\exp\{V_i(j; \bar{w} + u_i, T) + \mu_0 D_0(j \in M) + \sum_{g=1}^G \mu_g D_g(j \in M_g)\}}{\sum_k \exp\{V_i(k; \bar{w} + u_i, T) + \mu_0 D_0(k \in M) + \sum_{g=1}^G \mu_g D_g(k \in M_g)\}},$$

and the probability that individual i is matched to a market job as

$$(17) \quad \pi_i(T, \bar{w}, \mu_0(\bar{w}), \dots, \mu_G(\bar{w})) \equiv \sum_{j \in M} \frac{\exp\{V_i(j; \bar{w} + u_i, T) + \mu_0 D_0(j \in M) + \sum_{g=1}^G \mu_g D_g(j \in M_g)\}}{\sum_k \exp\{V_i(k; \bar{w} + u_i, T) + \mu_0 D_0(k \in M) + \sum_{g=1}^G \mu_g D_g(k \in M_g)\}}$$

The equilibrium conditions for a reform R in the finite, infinite and zero elasticity are respectively:

$$(18) \quad \begin{aligned} \sum_i \pi_i(R, \bar{w}_R, \mu_0(\bar{w}_R), \dots, \mu_G(\bar{w}_R)) &= J(\bar{w}_R) \\ \sum_i \pi_i^g(R, \bar{w}_R, \mu_0(\bar{w}_R), \dots, \mu_G(\bar{w}_R)) &= J_g(\bar{w}_R), g = 1, \dots, G \end{aligned}$$

$$(19) \quad \begin{aligned} \sum_i \pi_i(R, \bar{w}, \mu_{0R}, \dots, \mu_{GR}) &= He^{\mu_{0R}} \\ \sum_i \pi_i^g(R, \bar{w}, \mu_{0R}, \dots, \mu_{GR}) &= A_g He^{\mu_{0R} + \mu_{gR}}, g = 1, \dots, G. \end{aligned}$$

$$(20) \quad \begin{aligned} \sum_i \pi_i(R, \bar{w}_R, \mu_0(\bar{w}), \dots, \mu_G(\bar{w})) &= J(\bar{w}) \\ \sum_i \pi_i^g(R, \bar{w}_R, \mu_0(\bar{w}), \dots, \mu_G(\bar{w})) &= J_g(\bar{w}), g = 1, \dots, G. \end{aligned}$$

3.2. Couples

When analyzing the simultaneous labour supply decisions of married couples we might want to distinguish the choice set available to males (M) and females (F). For $S = F$ or M , expression (6) is generalized as follows:

$$(21) \quad \mu_{0S} = \ln\left(\frac{J_S}{H_S}\right).$$

² More generally we could specify job-specific wage rates.

We then specify gender-specific labour demand functions:

$$(22) \quad J_S = J_S(\bar{w}_F, \bar{w}_M).$$

Expressions (21) and (22) imply a mapping such as:

$$(23) \quad \mu_{0S} = \mu_{0S}(\bar{w}_F, \bar{w}_M).$$

Let us define $\pi_{iS}(T, \bar{w}_F, \bar{w}_M, \mu_{0F}(\bar{w}_F), \mu_{0M}(\bar{w}_M))$ as the probability that the partner of gender S in couple i works. Then the equilibrium conditions are

$$(24) \quad \sum_i \pi_{iS}(T, \bar{w}_{FR}, \bar{w}_{MR}, \mu_{0F}(\bar{w}_{FR}), \mu_{0M}(\bar{w}_{MR})) = J_S(\bar{w}_{FR}, \bar{w}_{MR}), S = F, M$$

in the finite elasticity case,

$$(25) \quad \sum_i \pi_{iS}(T, \bar{w}_F, \bar{w}_M, \mu_{0FR}, \mu_{0MR}) = H_M e^{\mu_{0SR}}, S = F, M$$

in the infinite elasticity case, and

$$(26) \quad \sum_j \pi_{jS}(T, \bar{w}_{FR}, \bar{w}_{MR}, \mu_{0F}(\bar{w}_F), \mu_{0M}(\bar{w}_M)) = J_S(\bar{w}_F, \bar{w}_M), S = F, M$$

in the zero elasticity case.

3.3 Matching equilibrium and Micro-Macro modeling

The matching model developed by Dagsvik (2000) replaces the simple concept of equilibrium adopted in this note with the notion of stable matching. Our equilibrium is a special case of a stable matching where the number of realized matches is equal to the number of available jobs and to the number of people willing to work. More generally, however, we can have a stable matching that involves vacancies and unemployment. A complementary research line would consist in specifying the opportunity density as the result of production decisions, which would provide a link to the recent attempts to develop models that integrate behavioural microsimulation within a macroeconomic or general equilibrium framework.³

4. An empirical illustration

We illustrate the procedure outlined above with a model of labour supply of Italian couples and singles, estimated on a 1998 EUROMOD dataset.⁴ The present exercise accounts for equilibrium

³ A survey is provided by Colombo (2008). A different approach, somewhat closer to partial equilibrium, is developed by Creedy and Duncan (2001).

⁴ The model is described in Colombino (2009) and in Colombino et al. (2010). In these papers, however, the analysis is limited to couples, while in this note we also use the estimates for singles.

between the total number of jobs and the number of people willing to work, according to expressions (24), (25) and (26).

For gender $S = F, M$ we adopt the following empirical specification for expression (22):

$$(27) \quad J_S = K_S \bar{w}_S^{-\eta}$$

where K_S is a constant and $-\eta$ is the elasticity of labour demand. In this illustrative exercise we will use imputed values for $-\eta$, although in principle both K_S and $-\eta$ might be estimated together with the household preferences. Expression (23) therefore turns out to be as follows:

$$(28) \quad \mu_{0S}(\bar{w}_S) = \ln \left(\frac{K_S \bar{w}_S^{-\eta}}{H_S} \right)$$

Given J_S (observed or simulated under the current tax-transfer system), \bar{w}_S (the mean of the estimated wage function), the estimated μ_{0S} and an imputed value of η we can use expressions (27) and (28) to retrieve H_S and K_S .

We simulate the effects of three hypothetical reforms that replace the current income support policies:

Universal Basic Income (UBI): Every household receives an unconditional transfer equal to 75% of the poverty line;⁵

Wage Subsidy (WS): Wage rates receive a 10% subsidy as long as earnings are below the poverty line;

Wage Subsidy + Guaranteed Minimum Income (WS + GMI): The wage subsidy is complemented by a transfer equal to 50% of the poverty line (conditional on income below the poverty line).

The size of the transfer is calibrated according to an equivalence scale. The policies are financed by widening the tax base to include all incomes of any source and by increasing the top marginal tax rates so that the total net tax revenue is kept unchanged. The exercise requires running the model iteratively until the equilibrium conditions and the total net tax revenue constraint are simultaneously satisfied.⁶ We present the results obtained using six different procedures: no account taken of equilibrium conditions; equilibrium with demand elasticity set equal to 0, -0.5, -1.0, -2.0 and $-\infty$. Table 1 illustrates the results. For the three policies and for each value of η we report the social welfare effect, the percentage of winners, the percentage change in disposable income, the

⁵ The poverty line is defined as 1/2 of the median of the sample distribution of the (equalized) household income.

⁶ We used the STATA module Amoeba in order to perform to constrained simulation.

percentage change in female labour supply (hours) and the percentage change in the top marginal tax rate required to maintain fiscal neutrality.⁷ We observe remarkable differences in the results depending on the value of η . When the simulation is performed without accounting for equilibrium conditions (as in the common practice), the ranking (in decreasing order) based on social welfare is: UBI, WS, WS+GMI, Current. The ranking starts changing when we set $\eta = 1$: WS+GMI, WS, UBI, Current. For $\eta = 2$ all the reforms are still welfare improving but with a different ranking, where WS emerges as the best reform: WS, WS+GMI, UBI, Current. When we approach the limit of a perfectly elastic labour demand ($\eta = \infty$), we get a radical change in the ranking and WS remains the only welfare-improving reform: WS, Current, WS+GMI, UBI. It is interesting to note that the standard practice of ignoring the equilibrium conditions implicitly claims to adopt a perfectly elastic demand scenario; however, the consistent procedure under that scenario would be the simulation with $\eta = \infty$, which indeed produces very different results with respect to the no equilibrium simulation. A realistic interpretation of the results might suggest $\eta = 0$ or $\eta = 0.5$ as the short-run equilibrium and $\eta = 1$ as the long-run equilibrium. Table 1 also reports some evidence on the behavioural effects: female labour supply (male supply is barely affected) and disposable income, which resumes a complex interplay between changes in labour supply, equilibrium wages, tax rates and transfers. For example, UBI induces a sensible reduction in female labour supply; its reflection upon disposable income is probably moderated by the universal transfer itself and by an increase in the equilibrium wage rates until we get to the case with $\eta = \infty$, when the wage rate does not change. WS clearly gives positive incentive to labour supply. This effect seems to be increasing with η , at least up to a certain point: when η approaches ∞ the incentive effect is counterbalanced by a smaller and smaller increase in equilibrium wage rates.

5. Conclusions

The procedure commonly used in simulating tax-transfer reforms with labour supply models adopting the “dummies refinement” is not consistent with the comparative statics interpretation of the policy simulation exercises. Based on a structural interpretation of the “dummies refinement”,

⁷ Social Welfare is measured as: (Average Individual Welfare) \times (1 – Gini index of the distribution of Individual Welfare). Individual Welfare is the money metric maximum expected utility (using as reference the worst-off household). This is similar to the so-called Sen Social Welfare index and it can be rationalized as a member of a rank-dependent social welfare indexes (e.g. Aaberge et al. 2006, 2010). The Table reports the change in Social Welfare divided by the average current disposable income. The percentage of winners is the percentage of households whose individual welfare increases.

we suggest an alternative procedure that is consistent with comparative statics. The procedure is illustrated with an empirical example where we simulate the effects of various hypothetical reforms of the income support mechanisms in Italy. The exercise shows how the results are affected depending on whether the equilibrium conditions are taken into account and on which value is imputed to the labour demand elasticity. On the one hand, it is somewhat reassuring that the results of the no equilibrium simulation are rather close to those obtained with the equilibrium procedure as long as η is below 1. On the other hand, it is worthwhile noting that the common practice of not accounting for equilibrium adjustment of the wage rates is usually interpreted as a perfectly elastic demand scenario (i.e. constant equilibrium wage rates). This interpretation however is not correct: indeed the simulation performed under the correctly specified scenario with perfectly elastic demand produces results that are radically different from those produced by the no equilibrium simulation.

Table 1. Simulated effects of the reforms. Alternative treatment of equilibrium conditions

| | | Welfare Gain (%) | Winners (%) | Change in disposable income (%) | Change in top marginal tax rate (%) | Change in female labour supply (%) |
|---------------------------------|----------|-------------------------|--------------------|--|--|---|
| No equilibrium | UBI | +1.3 | 57 | -1.0 | 23 | -2.0 |
| | WS | +1.2 | 71 | +0.5 | 7 | +0.4 |
| | WS + GMI | +0.6 | 56 | -1.2 | 13 | -1.1 |
| <hr/> | | | | | | |
| Equilibrium ($\eta = 0.0$) | UBI | +1.4 | 58 | -0.7 | 23 | -2.0 |
| | WS | +0.8 | 68 | +0.7 | 7 | +0.4 |
| | WS + GMI | +1.4 | 56 | +0.5 | 13 | -1.1 |
| <hr/> | | | | | | |
| Equilibrium ($\eta = 0.5$) | UBI | +1.3 | 58 | +0.0 | 23 | -1.9 |
| | WS | +1.2 | 70 | +1.6 | 7 | +0.5 |
| | WS + GMI | +0.5 | 57 | +0.7 | 13 | -1.0 |
| <hr/> | | | | | | |
| Equilibrium ($\eta = 1.0$) | UBI | +1.0 | 54 | -0.2 | 23 | -2.0 |
| | WS | +1.7 | 70 | +1.6 | 7 | +0.6 |
| | WS + GMI | +2,1 | 70 | -0.1 | 13 | -0.9 |
| <hr/> | | | | | | |
| Equilibrium ($\eta = 2.0$) | UBI | +0.6 | 45 | -0.3 | 23 | -2.2 |
| | WS | +2.7 | 71 | +1.7 | 7 | +0.8 |
| | WS + GMI | +1.9 | 57 | +0.7 | 13 | -0.6 |
| <hr/> | | | | | | |
| Equilibrium ($\eta = \infty$) | UBI | -7.9 | 13 | -1.8 | 23 | -4.9 |
| | WS | +0.4 | 65 | +0.4 | 7 | +0.3 |
| | WS + GMI | -0.5 | 47 | -0.6 | 13 | -1.2 |

References

- Aaberge, R., J.K. Dagsvik and S. Strøm (1995): Labour Supply Responses and Welfare Effects of Tax Reforms, *Scandinavian Journal of Economics*, 97(4), 635-659.
- Aaberge, R., Colombino, U. and S. Strøm (1999): Labour Supply in Italy: An Empirical Analysis of Joint Household Decisions, with Taxes and Quantity Constraints, *Journal of Applied Econometrics*, 14(4), 403-422.
- Aaberge, R. and U. Colombino (2006): Designing Optimal Taxes with a Microeconomic Model of Labour Supply”, IZA Discussion Paper n. 2468.
- Aaberge, R. and U. Colombino (2010): Accounting for Family Background when Designing Optimal Income Taxes: A Microeconomic Simulation Analysis, *Journal of Population Economics*, DOI: 10.1007/s00148-010-0331-y.
- Colombino, U. (2009): Evaluating Alternative Basic Income Mechanisms: A Simulation for European Countries, Discussion Papers 578, Research Department, Statistics Norway.
- Colombino, U., M. Locatelli, E., Narazani and C. O’Donoghue (2010): Alternative Basic Income Mechanisms: An Evaluation Exercise with a Microeconomic Model, *Basic Income Studies*, 5(1).
- Colombo, G. (2008): Linking CGE and Microsimulation Models: A Comparison of Different Approaches, ZEW Discussion Paper No. 08-054.
- Creedy, J. and A. Duncan (2001): Aggregating Labour Supply and Feedback Effects in Microsimulation, IFS WP01/24.
- Dagsvik, J. (1994): Discrete and Continuous Choice, Max-Stable Processes, and Independence from Irrelevant Attributes, *Econometrica*, 62(5) 1179-1205.
- Dagsvik, J. (2000): Aggregation in Matching Markets, *International Economic Review*, 41(1), 27-57.

Dagsvik, J. and S. Strøm (2006): Sectoral Labour Supply, Choice Restrictions and Functional Form, *Journal of Applied Econometrics* 21(6): 803–826.

Kalb, G. (2000): Accounting for Involuntary Unemployment and the Cost of Part-Time Work, Working Paper No. BP-35, Centre of Policy Studies, Monash University, Australia.

Kornstad, T. and T. Thoresen (2007): A Discrete Choice Model for Labor Supply and Childcare, *Journal of Population Economics*, 20(4), 781-803.

Van Soest, A. (1995): Structural Models of Family Labour Supply, *Journal of Human Resources*, 30(1), 63-88.