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CARLA MARCHESE, SIMONE MARSIGLIO, FABIO PRIVILEGGI and GIOVANNI B. RAMELLO

Department of Economics and Statistics "Cognetti de Martiis"
Campus Luigi Einaudi, Lungo Dora Siena 100/A, 10153 Torino (Italy)
www.est.unito.it

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Endogenous Recombinant Growth through Market Production of Knowledge and Intellectual Property Rights*

Carla Marchese† Simone Marsiglio‡ Fabio Privileggi§
Giovanni B. Ramello¶

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Abstract

We analyze the relationship between economic growth, knowledge production and intellectual property rights. Economists and historians underline different aspects as possible causes of knowledge accumulation; the former stress the role of incentive mechanisms while the latter the autonomous progress of science. We construct a unified theory allowing for the presence of markets and the autonomous accumulation of knowledge by introducing intellectual property right policies in an endogenous recombinant growth model. In this framework a benevolent government should reallocate resources from the final to the knowledge production sector and implement a tax-subsidy scheme in order to correct for the inefficiencies generated by the process. We characterize the (asymptotic) steady state equilibrium, and some properties of the transitional path. We show that if certain conditions are met, then the economy will converge to its (asymptotic) balanced growth path, and along such a path growth will be independent of the government policy; conversely, transition dynamics and the capital to knowledge ratio are affected by the choice of the tax-subsidy parameter. We then quantitatively analyze the effect of different policy interventions on welfare, and show that welfare is increasing in the policy parameter and a strictly positive policy level may be required to avoid stagnation.

JEL Classification Numbers: K11; L16; L43; O31; O34; O41

Keywords: Endogenous Recombinant Growth; Intellectual Property Rights; Knowledge Production; Welfare

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†Institute *POLIS - DiGSPES*, Università del Piemonte Orientale *Amedeo Avogadro*, Alessandria (Italy); carla.marchese@unipmn.it

‡School of Business, *James Cook* University, Cairns (QLD, Australia); simone.marsiglio@jcu.edu.au

§Dept. of Economics and Statistics "Cognetti de Martiis", Università di Torino, Lungo Dora Siena 100 A, 10153 Torino (Italy). Phone: +39-011-6702635; fax: +39-011-6703895; e-mail fabio.privileggi@unito.it

¶Institute *POLIS - DiGSPES*, Università del Piemonte Orientale *Amedeo Avogadro*, Alessandria (Italy); giovanni.ramello@unipmn.it

1 Introduction

Technological progress is a crucial ingredient in explaining economic growth. In standard neoclassical growth models technological progress is considered as a pure public good, non rival and non excludable, and its production is not analytically described, thus basically consisting of a black box. After the pioneering work of Romer (1987, 1990), the endogenous growth literature has increasingly pondered the inner dynamics of the technological progress involving knowledge advances and the role that partial excludability, by means of secrecy or intellectual property right (IPR) systems, can play. At the basis of this change of perspective lies the idea that also knowledge accumulation could be explained from the economic point of view, since in a decentralized market economy the aforementioned idiosyncratic incentive mechanisms play a basic role in explaining why economic factors, such as labor or human capital or resources in general, can be shifted from other uses and conveyed to produce knowledge.

On the other side, the literature on innovation also alleges that appropriation opportunities – implemented notably via intellectual property rights – which entitle the right holder an exclusive, monopolistic right over a fragment of knowledge, are primarily devoted to avoid the market failure arising from the free-riding problem in the presence of public goods. This interpretation is highly convenient for economic theory, as it likens knowledge production to standard manufacturing processes. The right holder is thus represented as the innovator/entrepreneur aiming at maximizing his profit, under the assumption that *"he who has no hope that he shall reap, will not take trouble to sow"* (Bentham, 1839, p. 31).

Such a stylization, nevertheless, raises several concerns. Since a monopoly is unable to appropriate all social surplus, in general it does not supply the optimal incentive to inventors and authors, while different arrangements preserving the public good nature of knowledge, such as rewards systems administered by the government,[1] seem to a large extent better equipped to attain optimality (Arrow, 1962; Shavell and van Ypersele, 2001). Further critical concerns relate to the incremental nature of knowledge production that recognize its role as an input; in such a case the above-cost pricing that monopoly entails will equally increase the cost of knowledge and the total incentive effect will be discounted by the increase in production costs. On the whole, the cost increase effect[2] yields ambiguous results both on social welfare and eventually on innovator profit (Scotchmer, 1991; Bessen and Maskin, 2009).

So far all the economic arguments reviewed are treating knowledge as a good or a sum of goods characterized by divisibility. By contrast, a critical approach that is gaining attention points out that knowledge is not just an intangible good or resource, defined and delimited like standard goods produced and exchanged on the markets, but a dynamic entity and a cognitive tool pertaining to social groups.[3] It is renewed through sharing among individuals, which is thus an indispensable feature for creative activity (Ramello, 2005, 2011). In fact knowledge for longtime was governed by appreciably open exchange mechanisms different from the market (mainly communication) and relying heavily on the sharing process. Also today many cultural

[1]Public funding in various forms does play today an important and not substitutable role in stimulating innovations, as it counts for at least one third of gross domestic R&D expenditures in many developed economies, US included (OECD, 2006).

[2]Besides the direct cost increase connected with the higher price of knowledge monopolized inputs (Scotchmer, 1991) there is an indirect one connected with transaction costs incurred for searching, contracting and buying the licenses for all the dispersed fragments of proprietary knowledge needed as inputs. When there are complementary inputs an adverse outcome, the so called 'tragedy of anticommons' can arise (Heller and Eisenberg, 1998; Ziedonis, 2004).

[3]Stiglitz (1999) on p. 315 reckons that *"Every innovation makes use of previously accumulated knowledge – it draws on the global commons of pre-existing knowledge"*.

contents and expressions are produced through social interactions, for motivations other than profit (pleasure of acquiring fame or of participating in a social network, pleasure deriving from individual or collective creative effort) or their links with the market are far and indirect (*e.g.*, expected increases in remuneration from related activities). These forms of knowledge production can involve contents that span from the basic components of culture (language, cultural heritage, information as part of the social capital) to the open source supply of new technological devices.[4]

The role of economic motivations in explaining knowledge advances is equally questioned by historians,[5] who emphasize the autonomous progress of science. Several historic studies show that discoveries in a particular field are largely related to the improvements in the scientific and engineering knowledge in that field (Rosenberg, 1976), or that path breaking innovations in specific areas, as computers, are largely determined by the pioneering role played by certain talented individuals not moved by profit motives (Ceruzzi, 2003). Recently, also studies in organizational economics, innovation and management have pointed out how not all types of innovations are driven by the profit principle. For example, Carlsson et al. (2009) distinguish between academic and industrial research, where the former is mainly focused on theoretical subjects and therefore has not immediate economic value per se,[6] but is essential in applied research, which is intrinsically associated with a clear economic value. This is confirmed also by Packalen and Bhattacharya (2012), who find that patents granted in the US increasingly quote scientific publications, and such citations are more frequent in more innovative patents.

Therefore, following the intuition of Ramello (2005, 2008, 2011), it is interesting to develop a model of economic growth that combines the role of market with that of the aforementioned dynamics, specifically recognizing that the stock of knowledge can grow endogenously with respect to itself, and not only respond to standard incentives provided by the economy, as maintained by most of the extant endogenous growth literature with a few exceptions.[7] Weitzman (1998) made an early step in this direction by assuming that the knowledge stock is endowed with some natural direct germinative property, which can be either fostered or hampered by the externalities stemming from the economy. In Weitzman's model, however, knowledge production is centralized. While maintaining the basic framework of the Weitzman's model, we depart from it by introducing a decentralized system for knowledge production under the protection of patents, somehow reconciling the Weitzman's view with the endogenous growth literature. Specifically, we assume that the government drives knowledge advances by buying newly produced useful ideas by innovative firms which in turn exploit already available ideas stemming for free from social non-market interactions. The government then releases new ideas for free to the firms producing final consumption goods. This institutional setting seems to be consistent with some experiences documented by economic history. In fact, governments have actually sometimes bought patents, and proposals have been advanced for adopting systematically such an approach.[8]

We aim at finding out the basic effects on economic growth of a simplified IPR system, in which the newly created knowledge legally owned by the innovator, *e.g.*, by means of a

[4]Such innovations can even occur at a sustained rhythm, as, *e.g.*, in software development (Frischmann, 2012).

[5]See also the references in Acemoglu (2009), pp. 414-417, and in Moser (2014).

[6]For the role of intrinsic motivations as opposite to economic incentives in academic research see also Aghion et al. (2008) and the references quoted therein.

[7]See, *e.g.*, Stiglitz (2014), who assumes that additions to knowledge depend positively on the pool of knowledge that is in the public domain, which is increased by non patented innovations and decreased by the "enclosures of the mind" arising from the IPR system.

[8]On this issue see for example p. 528 in Shavell and van Ypersele (2001).

patent, is bought by a benevolent government and then made available for free – *i.e.*, in the public domain – to everyone. Hence in our scenario the drawbacks caused by standard forms of monopolistic exploitation of IPR do not arise. However, the findings show that, at least in the short and mid-run, the appropriability via IPR still involves inefficiencies, due on one hand to price distortions, and on the other one to the waste of resources in administrative and legal expenditures for avoiding infringements. Moreover the IPR system can hamper the takeoff of the growth process in economies lacking sufficient initial resources, and has adverse effects on the capital to knowledge ratio, which is distorted in favor of physical capital. However, the power of the germinative property of knowledge implies that asymptotically economies with this IPR system converge to a socially efficient outcome in terms of growth.

The paper proceeds as follows. In Section 2 we shortly recall the recombinant growth pattern of knowledge in the continuous-time, Ramsey-type version of Weitzman's (1998) model developed by Tsur and Zemel (2007), who refer to a centralized system in which knowledge production is optimally decided and run by a regulator. In Section 3 we introduce our model specification featuring a decentralized market for knowledge production based on an IPR system and the possibility of government intervention to contain inefficiencies by applying a certain tax-subsidy scheme. In Section 4 we discuss short and long-run equilibria, showing that if certain conditions are met the economy converges towards an asymptotic balanced growth path along which the growth rate is not affected by the government policy, while the transition equilibrium depends on the tax-subsidy parameter. Section 5 contains our quantitative results: we basically show here that active government intervention may be needed in order to avoid stagnation, and that welfare is increasing with the policy parameter. Section 6 comments on the policy implications of our model, while Section 7 as usual concludes. All mathematical technicalities involved in our simulations can be found in a companion paper by Privileggi and Marsiglio (2014).

2 A Brief Review of Recombinant Growth

One of the most imaginative and comprehensive representations of the knowledge production process is provided by Schumpeter (1934) through the recombinant innovation, where the core of the innovative process resides in successfully combining in new ways already existing resources. In a similar vein, Weitzman (1998) postulates that originally unprocessed ideas – 'seeds' in his jargon – are blended with all other ideas available in order to generate new hybrid seed ideas; a costly selection process permits in turn to extract from these a subset of fertile seed ideas that are again recombined with all the existent fertile ideas to produce yet new hybrids, and so on. Such a process occurs indefinitely, generating knowledge growth. This mechanism produces what Jones (2005) labels as 'standing on the shoulders effect', meaning that the discovery of new ideas increases the possibility of discovering new ideas in the future. The number of newly generated successful ideas in his discrete time framework defines a recombinant expansion process of second-order through which the stock of knowledge has the potential of growing faster than exponentially, that is, at an increasing rate of growth (Lemma on p. 338 in Weitzman, 1998). However, because the hybridization process of seed ideas necessarily also consumes an amount of physical resources, potentially explosive growth is precluded by physical constraints; precisely, scarcity of resources. As a matter of fact, Weitzman shows that knowledge actually grows at some bounded positive rate, thus reconciling his theory with standard endogenous growth models, suggesting that the growth rate of GDP in real economies should be bounded as well (Barro and Sala-i-Martin, 2004; Aghion and Howitt, 2009; Acemoglu, 2009). Weitzman's recombinant dynamic also captures in a highly simplified and thus tractable way some relevant historical aspects (as innovations were actually often obtained by combining existing technologies) and rationalizes a likely potential similarity between economic and natural systems (Safarzynska and van den Bergh, 2011).

Tsur and Zemel (2007) simplify the second-order discrete-time dynamics associated to the production of new ideas by reformulating the model in a continuous-time Ramsey-type growth framework, in which the amount of resources employed in knowledge production, J, is no longer a predefined share of national income, as in Weitzman – who analyzes a Solow-type model – but becomes a variable to be optimally chosen by a benevolent regulator. In continuous-time the new knowledge production function is defined as

$$\dot{A}(t) = H(t)\,\eta\left[\frac{J(t)}{H(t)}\right], \tag{1}$$

where $\dot{A}(t)$ denotes the derivative of the stock of knowledge at instant t, $A(t)$, with respect to time t, $H(t)$ represents the number of hybrid seed ideas, and $\eta(\cdot)$ is the probability of obtaining a successful idea from each hybridization (matching) satisfying the following assumption, as in Weitzman (1998).

A. 1 *The success probability function* $\eta : \mathbb{R}_+ \to [0,1]$ *is independent of time and is such that* $\eta'(x) > 0$, $\eta''(x) < 0$, $\eta(0) = 0$ *and* $\lim_{x\to\infty} \eta(x) \leq 1$*; moreover, it will be assumed that* $\eta'(0) \equiv \lim_{x\to 0^+} \eta'(x) < +\infty$.

The necessary smoothness assumption required in a continuous-time setting implies that $\dot{A}(t)$ has the same value both while looking forward to the new output – equation (1) – and while looking backward, *i.e.*, to the formation of seed ideas, given by

$$H(t) = C'_m[A(t)]\,\dot{A}(t), \tag{2}$$

where $C'_m(A)$ denotes the derivative of the number of different combinations of m elements as a function of the stock[9] A (Tsur and Zemel, 2007; Privileggi, 2010, 2011); that is, within this approach both the seed production in (2) and the production of new ideas in (1) are referred (as a limit) to the same time instant. Hence, when (2) is substituted into (1), it turns out that the law of motion of knowledge at every instant is

$$\dot{A}(t) = \frac{J(t)}{\varphi[A(t)]}, \tag{3}$$

where

$$\varphi(A) = C'_m(A)\,\eta^{-1}\left[\frac{1}{C'_m(A)}\right], \tag{4}$$

is the expected unit cost of knowledge production as a function of the stock of knowledge A, converging to the constant $1/\eta'(0)$ as $A \to \infty$; note that $1/\eta'(0)$ is strictly positive under Assumption A.1.

Tsur and Zemel's benevolent regulator, by taxing household's income to finance knowledge production, can thus optimally choose J in order to foster knowledge growth. The optimization problem in Tsur and Zemel (2007) has two types of equilibria that can be characterized by means of curves in the knowledge-capital state space. Along one of these curves, that the authors name 'turnpike', growing economies evolve, while non growing economies converge to a point on a 'stagnation' curve. They show that sustained growth depends both on a condition relating

[9]Specifically: $C_m(A) = A!/[m!(A-m)!]$. For example, $C_2(A) = A(A-1)/2$.

the slopes of such curves and on a minimal endowment requirement; if these conditions hold, growing economies reach the turnpike and evolve along it thereafter, otherwise they stagnate forever.

While Tsur and Zemel's description is appropriate for a fully rational and perfectly informed benevolent decision maker[10] (who can actually oversee the adjustment of the seed provision to the desired production at each instant), the case in which knowledge is supplied within an IPR framework needs some further study, which is exactly the focus of our paper. In the following, in order to introduce a basic IPR system, we assume that the knowledge stock A is a public good that the government renders available for free in the public domain. The benevolent regulator decides how many resources to convey to the production of new knowledge, but these resources now represent the government expenditure devoted to buy the latter on the market. The seed ideas available at time t are still defined by (2), but we assume that they arise without the need of any specific compensation, thanks to the cultural and scientific debate that realizes the recombination of ideas. The R&D industry thus exploits H for free and combines it with J to produce knew knowledge $\dot{A}$.

3 The Model

Our model is a Ramsey-type model of growth with endogenous creation of knowledge. The economy is composed of households, firms and the government. Households receive interest income and a wage for the labor supplied to the production sector, pay a lump-sum tax, purchase a consumption good and choose how much to save. Moreover, they socially interact and combine the stock of existing ideas with the newly produced ones, thus producing seed ideas for free. There are two types of firms: one producing the final consumption good and one performing knowledge creation activities. The government collects taxes from households and invests the proceeds into knowledge creation; moreover, it resorts to a tax-subsidy scheme applied to the latter type of firms in order to correct the inefficiencies generated by the IPR system.

3.1 Households

As we focus mainly on the effects of IPR policies on growth patterns and welfare, in order to simplify the analysis, we shed population growth and, with no loss of generality, assume that the total number of households is constant and normalized to unity, and the size of the household labor supply is constant over time too: $L(t) \equiv 1$ for all t; therefore, we state the model directly in terms of per capita variables. We adopt the standard assumption that all households have an increasing and concave utility function, $u'(c) > 0, u''(c) < 0$ where c is consumption. Households' goal consists of choosing consumption in order to maximize their own lifetime utility under a capital accumulation constraint, and their initial capital endowment, $k(0)$:

$$\max_{[c(t)]_{t=0}^{\infty}} \int_0^{\infty} u[c(t)] e^{-\rho t} \, \mathrm{dt}$$
$$\text{subject to } \dot{k}(t) = r(t) k(t) + w(t) - c(t) - G(t),$$

[10]Similarly, La Torre and Marsiglio (2010) build a centralized model of optimal growth where the planner endogenously allocating human capital across different sectors ultimately determines the long-run growth rate of knowledge and of the whole economy.

where $\rho > 0$ is the rate of time preference, $r(t)$ the interest rate and $w(t)$ the wage households receive for renting capital, $k(t)$, and supplying labor respectively, $G(t)$ the lump-sum tax they pay, while they devote the income not consumed to capital accumulation.

Seeds ideas H arise for free from (costless) social interactions among households, in which new knowledge $\dot{A}$ released by the government is combined with the already existing knowledge stock $A(t)$ according to (2).

3.2 Firms

In the final consumption good sector firms (F-firms) are competitive and operate in a standard neoclassical framework: at each instant F-firm i employs capital and knowledge-augmented labor to produce a composite consumption good according to a constant returns to scale production function, $Y_i = F(K_i, AL_i)$, taking as given the interest rate, r, the labor wage, w, and the stock of knowledge, A. Because they all use the same technology and face the same market conditions, they also employ the same capital/labor ratio; hence the subscript i can be dropped and per capita output turns out to be $y = Y/L = F(k, A)/L$, where $k = K/L$. Moreover, as $F(\cdot, \cdot)$ is constant return to scale, all identical F-firms pay the production factors their marginal cost and earn zero profit.

Firms creating new knowledge ($R\&D$-firms) exploit the recombinant technology described in (1). Every $R\&D$-firm aims at profit maximization. The market for R&D is competitive. This is due to the fact that on the supply side there are many firms supplying a homogeneous good – new knowledge – while on the demand side there is a monopsonist, the government, which is benevolent and behaves as a price-taker. All $R\&D$-firms face the same market prices, produce the same output amount and use the same input combination. Thus, each firm uses a fraction of both the whole available seeds H and the overall resources J employed by the industry. In fact, even if seeds are free, the prohibition of patenting again the same research result already patented by some $R\&D$-firm granted by the IPR system implies that each firm *de facto* appropriates the seeds amount used in its activity. Hence, patents implicitly assign property rights on seeds, thus avoiding a potential tragedy of the commons.

It will be shown that under Assumption A.1 the $R\&D$-firms earn a positive profit. However, it is assumed that the expenditures needed for the patent's registration and for the legal support aimed at deterring potential challengers equal the profits of the holder of the patent;[11] hence the free entry condition is satisfied. Even if new knowledge $\dot{A}$ is sold in discrete amounts covered by patents, for simplicity we treat the whole industry output, $\dot{A}$, as a continuous function of time; its price arises as a market equilibrium, where total new knowledge supplied by the $R\&D$-firms equals the government's demand.

It is assumed that new knowledge can be directly transferred to production of final goods (through, *e.g.*, organizational and process innovations), while such transfer implies a rescaling to take into account the effectiveness in terms of output augmentation (Weitzman, 1998, p. 344), as will be clarified later on. The possibility of supplying directly knowledge contents to F-firms seems more realistic than that adopted in the input or product variety approach, which implies that innovations have to be embedded into machines or goods. Nowadays patents refer

[11] A large literature stresses that actually the IPR system entails huge administrative and legal costs (see, *e.g.*, Scotchmer, 2004, and the references quoted therein). Moreover it has been observed that incumbent patent holders are sometimes dissipating profits for endorsing preemptive strategies or for accumulating costly patent portfolios used as 'bargaining chips' (see, *e.g.*, Gilbert and Newbery, 1982; Hall and Ziedonis, 2001). For a model of economic growth à la Romer (1990) that internalizes the enforcement cost of patents see Eicher and García-Peñalosa (2008).

more and more to procedures, routines, marketing models, etc., *i.e.*, to dematerialized contents that F-firms can directly implement in their activity (Chantrel et al., 2012).

3.3 The R&D Market Equilibrium

All the R&D firms are symmetric and accordingly behave in the very same way; hence, we can simplify the analytical framework by considering the entire industry as a unique firm delivering the whole industry production. We thus assume that, like each single firm, the industry takes the price of new knowledge ψ and the amount available of factor H as given and maximizes its profit only with respect to J. This reproduces what the single firms do, as they too face price ψ for new knowledge, while they can use for free as much H as they need to obtain the patent. Hence, again dropping the time index for simplicity, the representative aggregate $R\&D$-firm maximizes its (instantaneous) profit, $\pi = \psi\dot{A} - J$, with respect to J. In view of (1), such profit can be rewritten as

$$\pi\left(H, J\right) = \psi H \eta\left(J/H\right) - J. \tag{5}$$

Strict concavity of $\eta\left(\cdot\right)$ and the fact that $\eta\left(\cdot\right) \leq 1$ [that is, $\psi H \eta\left(J/H\right)$ is bounded from above by ψH, which is independent of J] dictated by the smoothness assumption A.1 are enough for (5) to have a unique (interior) maximum J^* provided that $\left(\partial/\partial J\right)\pi\left(H, 0\right) = \psi\eta'\left(0\right) - 1 > 0$, that is, whenever

$$\psi > 1/\eta'\left(0\right). \tag{6}$$

We shall see that condition (6) is always satisfied under Assumption A.1 (see Proposition 2 below). Then, FOC on (5) with respect to J yields the optimal amount of physical resources employed in knowledge production for given H seed ideas and price ψ:

$$J^* = H\left(\eta'\right)^{-1}\left(1/\psi\right), \tag{7}$$

where $\left(\eta'\right)^{-1}$ denotes the inverse of the derivative of the probability η, which clearly exists as $\eta'\left(\cdot\right)$ is decreasing. By plugging J^* as in (7) into (1) we get the new knowledge level in our economy, as determined by the optimal $R\&D$-firm's input choice:

$$\dot{A} = H\eta\left[\left(\eta'\right)^{-1}\left(1/\psi\right)\right]. \tag{8}$$

Equation (8) determines the new knowledge amount provided that the regulator finds convenient to pay the price ψ for purchasing the new knowledge $\dot{A}$.

In order to explicitly compute ψ, we equate the $R\&D$-firm's revenues to the amount of resources, G, paid by the regulator for purchasing the newly created knowledge:

$$G = \psi\dot{A}. \tag{9}$$

Substituting ψ as in (9) into (8) yields

$$\dot{A} = H\eta\left[\left(\eta'\right)^{-1}\left(\dot{A}/G\right)\right],$$

which, after replacing H as in (2), leads to

$$\dot{A} = G\eta'\left\{\eta^{-1}\left[1/C_m'\left(A\right)\right]\right\}. \tag{10}$$

In view of (9), (10) can be rewritten as

$$\dot{A} = G/\psi\left(A\right), \tag{11}$$

where ψ

$$\psi(A) = \frac{1}{\eta'\{\eta^{-1}[1/C'_m(A)]\}} = (\eta^{-1})' \left[\frac{1}{C'_m(A)}\right] \tag{12}$$

provides an explicit formula for the expected price of knowledge production as a function of the stock of knowledge A paid by the regulator to $R\&D$-firms.

The law of motion defined by (11) is directly comparable to that defined by (3) for the first-best economy. Our first result is at the root of the differences between the Tsur and Zemel (2007) economy and the decentralized version with IPR.

Proposition 1

i) *For any given (finite) stock of knowledge, A, the price of knowledge production for the economy with decentralized R&D production, $\psi(A)$ defined in (12), is always larger than the cost $\varphi(A)$ defined in (4) borne by the social planner in the first-best economy:*

$$\psi(A) > \varphi(A) \qquad \text{for all } A < \infty. \tag{13}$$

ii) *However, when knowledge becomes abundant they converge to the same value:*

$$\lim_{A\to\infty} \psi(A) = \lim_{A\to\infty} \varphi(A) = \frac{1}{\eta'(0)}. \tag{14}$$

Proof. i) Using (12) and (4), condition (13) is equivalent to

$$(\eta^{-1})' [1/C'_m(A)] > C'_m(A)\, \eta^{-1}[1/C'_m(A)]. \tag{15}$$

Let

$$x = 1/C'_m(A); \tag{16}$$

as $A < \infty$ implies $C'_m(A) < \infty$, $x > 0$ holds true. Substituting (16) into (15) yields

$$(\eta^{-1})'(x) > [\eta^{-1}(x)]/x, \tag{17}$$

which holds true for all $x > 0$ under Assumption A.1.

ii) Condition (14) follows immediately by noting that

$$\lim_{A\to\infty} \psi(A) = \lim_{A\to\infty} (\eta^{-1})' [1/C'_m(A)] = (\eta^{-1})'(0) = 1/\eta'(0) = \lim_{A\to\infty} \varphi(A), \tag{18}$$

where the last equality is (5) in Tsur and Zemel (2007). ■

Proposition 1 states that, while the price of knowledge production in a decentralized R&D economy is always larger than the cost borne by the social planner in the first-best economy, as the stock of knowledge gets larger and larger, both converge to the same value. For an intuition about this result note that the wedge between $\psi(A)$ and $\varphi(A)$ is due to the scarcity rent accounted for by the $R\&D$-firms with reference to the seeds H that they appropriate. This rent, however, shrinks and virtually disappears as seeds become relatively abundant with respect to J.

Proposition 2 *Condition (6) always holds under Assumption A.1. Moreover, the instantaneous optimal profit of the representative R&D-firm is given by*

$$\pi^*(A) = [\psi(A) - \varphi(A)]\, \dot{A} \tag{19}$$

and is strictly positive for all $A < \infty$.

Proof. By Assumption A.1 $\psi(A) = (\eta^{-1})' [1/C'_m(A)]$ is decreasing in A, so that, by (18), $\psi(A) > \lim_{A\to\infty} \psi(A) = 1/\eta'(0)$ for all $A < \infty$; hence, (6) holds. Plugging the optimal effort J^* as in (7) into the profit defined as a function of (A, J), $\pi(A, J) = \psi(A)\dot{A} - J$, yields:

$$\begin{aligned}
\pi^*(A) &= \psi(A)\dot{A} - J^* \\
&= \psi(A)\dot{A} - H(\eta')^{-1}[1/\psi(A)] \\
&= \psi(A)\dot{A} - C'_m(A)\dot{A}(\eta')^{-1}[1/\psi(A)] \\
&= \psi(A)\dot{A} - C'_m(A)\dot{A}(\eta')^{-1}\left\{\eta'\left[\eta^{-1}(1/C'_m(A))\right]\right\} \\
&= \psi(A)\dot{A} - C'_m(A)\dot{A}\left\{\eta^{-1}[1/C'_m(A)]\right\} \\
&= [\psi(A) - \varphi(A)]\dot{A},
\end{aligned}$$

where in the third equality we used (2), in the fourth we used the first equality in (12), and in the sixth we used (4). By Proposition 1 (i) $[\psi(A) - \varphi(A)]\dot{A} > 0$ for all $A < \infty$ when there is knowledge growth (*i.e.*, a positive amount of new knowledge, $\dot{A}$, is produced and sold to the regulator). ■

Proposition 2 states that the *R&D*-firms obtain a positive profit, even if they operate in a competitive market. In fact their supply price incorporates a scarcity rent pertaining to the H factor, which these firms treat as fixed and appropriate for free thanks to the IPR system. Due to the fragmentation of the knowledge production process, the *R&D*-firms are not able to take into account the externalities produced by their activity, which contribute to the seeds' formation. Such externalities imply that actually the H supply is not fixed; it adapts instead at each instant to the knowledge evolution according to (2). Eventually, when seeds H become abundant, the rent incorporated into the market price shrinks and the over-cost pricing in the decentralized system fades out. Thus, it turns out that the basic unwanted effects of an IPR system are: i) from the distributive point of view, the private appropriation by the IPR holder of a rent pertaining to a public good, *i.e.*, new knowledge items and ii) a price of new knowledge larger than its marginal social cost, as the positive externalities generated by the new knowledge production are not taken into account. Proposition 1 states that these drawbacks tend to fade away in mature economies, where the stock H is at any rate already abundant. Poor countries aiming at starting a growth process anew, instead, are likely to find the IPR system more costly. This implies that the IPR system raises problems not only in terms of efficiency, but also of equity.[12]

Note also that in our model a quite realistic feature of the Weitzman model, not considered by Tsur and Zemel, *i.e.*, the scarcity of seeds as a source of decreasing returns to J in knowledge production, fading away only in the long run, is recovered, albeit in a different framework. While in the Weitzman's model this arises out of a short term effect within a discrete second order growth process, in our decentralized case it appears as a consequence of the appropriation of a public good via IPR and of a drawback in the *R&D*-firms' decision process due to the lack of consideration of the externalities of new knowledge production.

3.4 Government

In our stylized framework the government levies lump-sum taxes on households in order to buy and then to distribute for free across the economy newly produced ideas, maintaining a

[12] Ramello (2008) notes that, because of indivisibility characterizing the knowledge production, there is a unique link between efficiency and equity, and the intensity of appropriation via IPR can thus negatively affect both at the same time.

balanced budget at any point in time. Additionally, the government can correct the market failure arising in the innovative sector and restore optimality. The government basically resorts to a tax-subsidy scheme in order to improve efficiency; as the problem is that the price of the private provision of knowledge via IPR is larger than the social cost that would occur in a centralized regime, the corrective intervention should consist in a subsidy to the knowledge production, financed through a non distortionary tax on the profit of the *R&D*-firms.[13]

More specifically, as long as the government aims at implementing the efficient knowledge production that would arise under centralization, $\dot{A} = J/\varphi(A)$ as defined in (3), it should increase its own demand – with respect to the level that the latter would take given the price $\psi(A)$ – so that

$$\frac{G+d}{\psi(A)} = \frac{J}{\varphi(A)} = \dot{A}, \tag{20}$$

where $G < J$ represents the R&D financing under decentralization according to (9), and d is the subsidy needed to correct the market failure. Assuming that the government can levy taxes directly on *R&D*-firms' profits at a rate $0 \leq \tau \leq 1$, according to (19) we can set[14] $d = \tau[\psi(A) - \varphi(A)]\dot{A}$, so that, by (20), $G + d = G + \tau[\psi(A) - \varphi(A)]\dot{A} = \psi(A)\dot{A}$, which immediately yields the law of motion of knowledge that will specify our version of the recombinant growth model:

$$\dot{A} = \frac{G}{\phi_\tau(A)}, \tag{21}$$

where

$$\phi_\tau(A) = \tau\varphi(A) + (1-\tau)\psi(A), \tag{22}$$

with $\varphi(A)$ and $\psi(A)$ defined in (4) and (12) respectively.

Hence, in our setting, the unit cost of knowledge production as a function of the stock of knowledge A, $\phi_\tau(A)$, turns out to be a convex linear combination of the analogous costs under centralization (corresponding to the Pareto optimal equilibrium studied by Tsur and Zemel, 2007), $\varphi(A)$, and under decentralization, $\psi(A)$, depending on the tax parameter τ. Conversely, $(1-\tau)$ indicates the net rent appropriability provided by the patent system. If the government, by setting $\tau = 1$, were able to fully expropriate the *R&D*-firms' profits, the same first-best knowledge growth path observed under centralization would be reached through a redistributive process based on a tax-subsidy scheme. While in principle this would fully fix the efficiency problem, such implementation is impossible because it requires an exact assessment and expropriation of the firm's profits, a task generally deemed outside the reach of the government in a decentralized economy, where asymmetry of information characterizes the relationship between the government and the private sector. On the other hand, $\tau = 0$ implies fully decentralized, inefficient knowledge production where, however, the rent appropriation by the patent holder is maximal.

The main focus of this paper is on partial implementations of such a scheme. Specifically, we aim at studying how the economy as a whole reacts to different IPR policies (different values of the parameter $0 \leq \tau < 1$, which is assumed to be constant over time) – conversely,

[13]Note that our economy presents two inefficiencies (a public good and an externality), so that two policy instruments (the tax on households and the tax-subsidy mechanism on *R&D*-firms) are needed in order to restore optimality as in standard theory of economic policy (Tinbergen rule). The peculiarity of such inefficiencies is related to the double nature of knowledge, as input (seeds generate an externality) and output (the recombinant process generates new knowledge, a public good).

[14]Note that, in order to be non distortionary, the tax should be levied as a share of pure profits. As long as the latter are cut by taxation, expenditures aimed at discouraging those who might contend their IPR – which previously where assumed to exhaust profits – can be cut accordingly.

distinct private appropriability degrees, as measured by $(1-\tau)$ – in terms of long-run equilibria and transition time-path trajectories of the main variables involved, like stock of knowledge, physical capital, output and consumption, their growth rates and welfare.

3.5 Model Specification

For a given value of the policy parameter $0 \leq \tau < 1$, under general equilibrium all markets always clear and our economy can be summarized via a welfare maximization problem where, according to the utilitarian approach, the government as a social planner chooses the optimal amount G to be employed for buying new knowledge and the optimal level of consumption c in order to maximize the representative household lifetime utility, given the laws of motion of knowledge [according to (21)] and capital, and the initial conditions $k(0) = k_0$ and $A(0) = A_0$:

$$\max_{[c(t),G(t)]_{t=0}^{\infty}} \int_0^{\infty} u[c(t)]\, e^{-\rho t}\, \mathrm{dt} \tag{23}$$
$$\text{subject to } \begin{cases} \dot{A}(t) = G(t) / \phi_\tau [A(t)] \\ \dot{k}(t) = F[k(t), A(t)] - c(t) - G(t). \end{cases}$$

with the additional constraints[15] $G(t) \leq y(t)$, $c(t) \leq k(t) + y(t)$, and usual non-negativity constraints. For simplicity, it is assumed that capital does not depreciate. Suppressing the time argument, the current-value Hamiltonian associated to (23) is

$$H(k, A, G, c, \lambda, \delta) = u(c) + \lambda [F(k, A) - G - c] + \delta \frac{G}{\phi_\tau(A)}, \tag{24}$$

where λ and δ are the costate variables associated with k and A respectively. Necessary conditions are:

$$u'(c) = \lambda \tag{25}$$

$$G = \begin{cases} 0 & \text{if } \delta/\phi_\tau(A) < \lambda \\ \tilde{G} & \text{if } \delta/\phi_\tau(A) = \lambda \\ F(k, A) & \text{if } \delta/\phi_\tau(A) > \lambda \end{cases} \tag{26}$$

$$\dot{\lambda} = \rho\lambda - \lambda F_k(k, A) \tag{27}$$

$$\dot{\delta} = \rho\delta - \lambda F_A(k, A) + \delta \frac{G\phi'_\tau(A)}{[\phi_\tau(A)]^2} \tag{28}$$

$$\lim_{t\to\infty} H(t)\, e^{-\rho t} = 0, \tag{29}$$

where $\tilde{G}$ in (26) will be defined by (41) in the next section.

Following Privileggi (2010, 2011, 2013), in the sequel we shall assume that only pairs of ideas can be matched[16] and that the probability of success is described by a hyperbolic function.

A. 2 $m = 2$ *and the success probability function is given by*

$$\eta(x) = \beta x / (\beta x + 1), \qquad \beta > 0. \tag{30}$$

[15] The assumption that $G(t) \leq y(t)$ for all $t \geq 0$ departs from Privileggi (2010, 2011) who allowed for the investment G in new knowledge production to be larger than output. Such assumption brings the model back to the original framework of Weitzman (1998) and Tsur and Zemel (2007), in which the social planner chooses how much of the total output should be saved to be invested in R&D activities.

[16] In principle, matching a fraction number of ideas may work in this setting; we chose however to follow the original interpretation of the Weitzman (1998) dynamics by assuming that m is a natural number.

It is immediately seen that $\eta(\cdot)$ as defined in (30) satisfies Assumption A.1 with $\eta'(0) = \beta$. Parameter β provides a measure of the '*degree of efficiency*' of the Weitzman matching process: the larger β the higher the probability of obtaining a new successful idea out of each (pairwise) matching of seed ideas. For $m = 2$, $C_2'(A) = (2A-1)/2$ and from (30) we get $\eta^{-1}(x) = x/[\beta(1-x)]$; using these into (4) and (12) we get

$$\varphi(A) = \frac{1}{\beta}\left(\frac{2A-1}{2A-3}\right) = \frac{1}{\beta}\left(1 + \frac{2}{2A-3}\right) \tag{31}$$

$$\psi(A) = \frac{1}{\beta}\left(\frac{2A-1}{2A-3}\right)^2 = \frac{1}{\beta}\left(1 + \frac{2}{2A-3}\right)^2, \tag{32}$$

which, when substituted in (22), yield the following expression for the unit cost of knowledge production:

$$\phi_\tau(A) = \varphi(A)\left[(1-\tau)\beta\varphi(A) + \tau\right] = \frac{1}{\beta}\left(\frac{2A-1}{2A-3}\right)\left[(1-\tau)\left(\frac{2A-1}{2A-3}\right) + \tau\right], \tag{33}$$

defined for $A > 3/2$ and $0 \leq \tau < 1$. Note that, thanks to (32), the first equality in (33) defines $\phi_\tau(A)$ as a function of the unit cost $\varphi(A)$ introduced by Tsur and Zemel (2007). It is immediately seen that, for each given τ, $\phi_\tau(A)$ is decreasing in A, while Proposition 1 implies that $\psi(A) \geq \phi_\tau(A) > \varphi(A)$ for all $A > 3/2$ and all $0 \leq \tau < 1$, and $\lim_{A\to\infty}\phi_\tau(A) = \lim_{A\to\infty}\varphi(A) = 1/\eta'(0) = 1/\beta > 0$.

A. 3 *Output is produced according to a Cobb-Douglas technology:*

$$y(t) = F[k(t), A(t)] = \theta[k(t)]^\alpha [A(t)]^{1-\alpha}, \qquad \theta > 0,\ 0 < \alpha < 1, \tag{34}$$

depending on aggregate capital, $k(t)$*, and knowledge-augmented labor,* $A(t)L(t)$*, for* $L(t) \equiv 1$.

In assuming decreasing returns to each factor taken alone we follow Weitzman (1998) and Tsur and Zemel (2007), thus departing from the standard approach which excludes decreasing returns to scale to knowledge.[17] Weitzman (1998, p. 344) notes that knowledge used as input in the production of final goods should be measured in units of "effective augmentation" of the output. The quantification of the available stock of knowledge thus might be different from that relevant from the point of view of the recombination process. Making the assumption of decreasing returns to scale to knowledge in production while using a unique scale of measurement of knowledge is thus equivalent to assuming that the suitable scale for the use in production should keep into account some loss of effectiveness, due, *e.g.*, to coordination difficulties, partial substitution of previously used results, etc.

Each output producing F-firm maximizes instantaneous profit by renting capital k and hiring labor L from the households, taking as given the capital rental rate r, the labor wage w and the stock of knowledge A. Since F-firms act competitively, it follows from A.3 that:

$$\theta\alpha(k/A)^{\alpha-1} = r. \tag{35}$$

A. 4 *The instantaneous utility function is CIES,*

$$u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}, \qquad \sigma \geq 1. \tag{36}$$

[17]The standard argument relies on the concept of replicability: to double the copies of a design you may need to double all the factors but you do not need to double the design. The idea is that once the right to use a certain blueprint or idea has been obtained (through purchase or for free), such a blueprint can be used in all the production sites.

4 Equilibria and Endogenous Growth

As in Tsur and Zemel (2007) and Privileggi (2010, 2011, 2013), three curves on the space (A, k) are useful for characterizing the solutions of the social planner problem in our regulated economy.

1. The locus satisfying $F_k(k, A) = F_A(k, A) / \phi_\tau(A)$, which is characterized by allocative efficiency in production. While both capital and knowledge have an opportunity cost in terms of renounced consumption – that is, both imply a rental cost equal to r – knowledge has a further direct cost corresponding to its price. Hence, the allocation efficiency condition in production,[18] namely marginal rate of technical substitution (MRTS) equal to the input price ratio, can be written as
$$\frac{F_k(k, A)}{F_A(k, A)} = \frac{r}{r\phi_\tau(A)}, \tag{37}$$
so that $F_k(k, A) = F_A(k, A) / \phi_\tau(A)$. Under Assumptions A.2, A.3 and using (33) for a given τ, it can be rewritten as a function of the only variable A:
$$\tilde{k}_\tau(A) = \frac{\alpha}{1-\alpha}\phi_\tau(A) A = \frac{\alpha}{\beta(1-\alpha)}\left(\frac{2A-1}{2A-3}\right)\left[(1-\tau)\left(\frac{2A-1}{2A-3}\right) + \tau\right] A. \tag{38}$$
We call the $\tilde{k}_\tau(A)$ curve in (38) the *(transitory) turnpike*.

2. The function $\tilde{k}_\tau(A)$ in (38) for large A becomes affine, defining the curve
$$\tilde{k}_\tau^\infty(A) = \frac{\alpha}{\beta(1-\alpha)}(A + 2 - \tau), \tag{39}$$
which is the asymptote of the turnpike $\tilde{k}_\tau(A)$ for $A \to \infty$. Note that $\tilde{k}_\tau(A)$ lies above $\tilde{k}_\tau^\infty(A)$ for all $A < \infty$, approaching $\tilde{k}_\tau^\infty(A)$ from above as A increases. We call $\tilde{k}_\tau^\infty(A)$ in (39) the *asymptotic turnpike*.

3. Finally, on the locus $\theta\alpha(k/A)^{\alpha-1} = \rho$ the marginal product of capital equals the individual discount rate, which, by (35), implies $r = \rho$. It can be written as a linear function of A:
$$\hat{k}(A) = (\theta\alpha/\rho)^{1/(1-\alpha)} A. \tag{40}$$
We call $\hat{k}(A)$ in (40) the *stagnation line*, as it defines the set of all possible pairs (k, A) to which the economy eventually converges when – perhaps after some transitory path – after a finite time resources stop being invested in R&D, $G(t) \equiv 0$ for $t > t_0 \geq 0$, so that the stock of knowledge A remains steady and thus long-run growth is ruled out.

Differentiating $\tilde{k}_\tau(A)$ in (38) with respect to time and substituting into both equations forming the dynamic constraint of problem (23) yields
$$\tilde{G}(t) = [\tilde{y}_\tau(t) - \tilde{c}(t)] \frac{\phi_\tau[A(t)]}{\tilde{k}'_\tau[A(t)] + \phi_\tau[A(t)]}, \tag{41}$$
where $\tilde{y}_\tau(t) = \theta\left\{\tilde{k}_\tau[A(t)]\right\}^\alpha [A(t)]^{1-\alpha}$. Condition (41) establishes a relationship between the optimal investment in R&D, $\tilde{G}(t)$, as a function of the other control variable, the optimal consumption $\tilde{c}(t)$, when the economy is constrained to grow along the curve $\tilde{k}_\tau(A)$ defined in (38); that is, in view of (26), when $\delta(t) / \phi_\tau[A(t)] = \lambda(t)$ holds.

[18] Note that full efficiency is not reached as long as the price of knowledge items is larger than the marginal cost; it is reached only at the limit, when the former converges to the latter.

Proposition 3

i) *For any policy parameter value $0 \leq \tau < 1$, a necessary condition for the economy to sustain long-run growth requires the stagnation line to lie above the asymptotic turnpike for sufficiently large A, that is, $(\theta\alpha/\rho)^{1/(1-\alpha)} > \alpha/[\beta(1-\alpha)]$ must hold, which can be written in terms of an upper bound for the discount rate,*

$$\rho < r_\infty, \tag{42}$$

where $r_\infty = \theta\alpha[\beta(1-\alpha)/\alpha]^{1-\alpha}$ defines the long-run capital rental rate. *Conversely, if $\rho \geq r_\infty$ the economy eventually reaches a steady (stagnation) point on the line $\hat{k}(A)$ in (40) corresponding to zero growth.*

ii) *Under (42), for a given tax-subsidy scheme $0 \leq \tau < 1$ and initial knowledge stock $A_0 > 3/2$ there is a corresponding threshold capital stock $k_\tau^{sk}(A_0) \geq 0$ such that whenever $k_0 \geq k_\tau^{sk}(A_0)$ the economy – possibly after an initial transition outside the turnpike – first reaches the turnpike $\tilde{k}_\tau(A)$ in a finite time, and then continues to grow along it as time elapses until the asymptotic turnpike $\tilde{k}_\tau^\infty(A)$ is reached in the long-run. Along $\tilde{k}_\tau^\infty(A)$ the economy follows an asymptotic balanced growth path (ABGP) characterized by a* common constant growth rate *of output, knowledge, capital and consumption given by*

$$\gamma = (r_\infty - \rho)/\sigma, \tag{43}$$

which is independent of the tax parameter τ. *Moreover, $G(t) > 0$ for all t, while, as $t \to \infty$, $G(t) < y(t)$ and the income shares devoted to investments in knowledge and capital, denoted by s_∞ and s_∞^k respectively, are constant and again independent of τ; they are given by*

$$s_\infty = (1-\alpha)\gamma/r_\infty \qquad \textit{and} \qquad s_\infty^k = \alpha\gamma/r_\infty. \tag{44}$$

If $k_0 < k_\tau^{sk}(A_0)$ the economy eventually stagnates.

Proof. As the unit price of knowledge production function in (33) behaves in the very same way as the unit cost function in (31), the proof of Proposition 3 exactly parallels that in Tsur and Zemel (2007) for the first-best case. The constants γ in (43) and income shares s_∞, s_∞^k in (44) are independent of τ because, by Proposition 1 (ii), $\lim_{A\to\infty}\phi_\tau(A) = \lim_{A\to\infty}\varphi(A)$. ■

Proposition 3 establishes that if (42) holds – *i.e.*, either households are patient enough or, given the impatience of households, economies are more productive (larger θ) and/or with more efficient R&D (larger β) – and k_0 is sufficiently high with respect to the initial knowledge stock, A_0, the economy grows along a turnpike path which, in the long-run, converges to an asymptotic balanced growth path (ABGP) with knowledge and capital growing at the same constant rate and with constant income shares devoted to knowledge and capital increase.[19] For a Cobb-Douglas economy the long-run income shares allocated to R&D and savings turn out to be proportional to the knowledge and capital shares in the production function.

Remark 1 *Along the ABGP, the necessary condition for growth (42), the long-run economic growth rate in (43) and the income shares in (44) turn out to be completely independent of the policy parameter τ; only the Skiba-type point $k_\tau^{sk}(A_0)$, the turnpike $\tilde{k}_\tau(A)$ in (38) and the asymptotic turnpike $\tilde{k}_\tau^\infty(A)$ in (39) depend on τ. Therefore, the growth behavior of the economy in the long run is not affected by whether new knowledge is being produced in a centralized manner or under decentralization by means of IPR (with or without corrective tax-subsidy interventions).*

[19]See Palivos et al. (1997) for the existence of BGP and ABGP equilibria.

Different turnpike behaviors due to different policy parameter values are characterized in the next proposition, which is a direct consequence of Proposition 1.

Proposition 4

i) *The turnpike curves are monotonic with respect to the tax rate τ for fixed A:*

$$\tau_1 < \tau_2 \quad \Longleftrightarrow \quad \tilde{k}_{\tau_1}(A) > \tilde{k}_{\tau_2}(A) \quad \text{for all } A < \infty$$
$$\tau_1 < \tau_2 \quad \Longleftrightarrow \quad \tilde{k}^{\infty}_{\tau_1}(A) > \tilde{k}^{\infty}_{\tau_2}(A) \quad \text{for all } A,$$

with $\tilde{k}_\tau(A)$ and $\tilde{k}^{\infty}_\tau(A)$ defined in (38) and (39) respectively;

ii) *on the turnpikes also outputs, $y_\tau(A) = \theta\left[\tilde{k}_\tau(A)\right]^{\alpha} A^{1-\alpha}$ defined in (34), and interest rates, $r_\tau(A) = \theta\alpha\left[\tilde{k}_\tau(A)/A\right]^{\alpha-1}$ defined in (35), are monotonic with respect to the tax rate τ for fixed A: $y_{\tau_1}(A) > y_{\tau_2}(A)$ and $r_{\tau_1}(A) < r_{\tau_2}(A)$ for $\tau_1 < \tau_2$ and fixed A.*

Proof. Part (i) of Proposition 4 follows immediately from Proposition 1 (i) and the definitions of $\tilde{k}_\tau(A)$ and $\tilde{k}^{\infty}_\tau(A)$ in (38) and (39); part (ii) follows from part (i) and the definitions (34) and (35).[20] ■

As in Privileggi (2010, 2011, 2013), two optimal regimes are possible:

1. zero R&D, $G(t) \equiv 0$, which, if maintained forever, eventually leads the economy to some steady state (stagnation point) on the line $\hat{k}(A)$, and
2. a path along the turnpike $\tilde{k}_\tau(A)$ – maybe started after a finite period of transition outside the turnpike – envisaging growth as time elapses and, if maintained forever, eventually leading to an ABGP along the asymptotic turnpike $\tilde{k}^{\infty}_\tau(A)$.

Proposition 4 states that economies with less public intervention – corresponding to smaller τ, *i.e.*, with more unfettered IPR regimes – require larger capital/knowledge and output/knowledge ratios in order to sustain growth; accordingly, the interest rate is smaller. The intuition about this result is that a more expanded IPR system (τ close to zero) distorts the capital/knowledge ratio in favor of capital, as the higher knowledge price induces an increase of the capital/knowledge ratio in order to satisfy the efficiency property in (37), that is, $MRTS =$ input price ratio.

The capital/knowledge ratio distortion in favor of capital involves a larger than optimal exploitation of physical inputs, a fact that might have negative consequences on the environment. Another distortion is that the smaller the corrective public intervention the higher the threshold capital stock required for starting the growth process, $k^{sk}_\tau(A_0)$, that is, the more demanding becomes the growth path in terms of initial wealth and consumer's patience. As will be shown through examples in Section 5 the IPR system can imply a notable difference in the opportunities available to different countries according to their initial conditions, allowing an easy takeoff only to the richest economies. The tax-subsidy scheme mitigates these effects, by bringing down the price of knowledge, at a level closer to the social marginal cost.

In order to study how a given economy reacts to different IPR policies (different values of $0 \leq \tau < 1$, assumed to be constant over time) we perform comparative dynamics by changing

[20] It can be shown that Proposition 4 holds under a general neoclassic production function satisfying the usual neoclassic assumptions recalled on p. 3462 in Tsur and Zemel (2007).

the value of parameter τ while keeping constant all other parameters' values according to Subsection 3.5. Specifically, we aim at 1) studying how the (unique) Skiba-type point $k_\tau^{sk}(A_0)$ on the turnpike changes for different τ-values and 2) comparing social welfare values yield by optimal trajectories corresponding to different tax-subsidy regimes for an economy starting from the same initial pair (k_0, A_0) in $t = 0$.

Focusing on the latter regime above, under the necessary growth condition (42) and if $k_0 \geq k_\tau^{sk}(A_0)$ it can be shown that the turnpike $\tilde{k}_\tau(A)$ is 'trapping', *i.e.*, the economy keeps growing along it after it is reached. In this scenario there are two types of transitions: one driving the system toward the turnpike starting from outside it, and another characterizing the optimal path along $\tilde{k}_\tau(A)$ after it has been entered. From Proposition 4 (i) we know that there exists only one turnpike that crosses the initial point (k_0, A_0), corresponding to a specific value for parameter τ; that is, there is one single value for τ such that the associated turnpike $\tilde{k}_\tau(A)$ satisfies $\tilde{k}_\tau(A_0) = k_0$. This implies that, if we are to compare optimal trajectories corresponding to different policies starting from the same initial condition (k_0, A_0), besides characterizing the optimal dynamics along their corresponding turnpikes we must also study their optimal dynamics outside such turnpikes for the initial time interval required to reach them.

5 Simulations and Welfare Analysis

In this section we present results based on a simulation whose details can be found in Privileggi and Marsiglio (2014). We assume the following values for the fundamentals parameters in our economy, which are common in the macroeconomic literature (see, *e.g.*, Mulligan and Sala-i-Martin, 1993, and Privileggi, 2010, 2011, 2013):[21]

$$\alpha = 0.5, \qquad \rho = 0.04, \qquad \theta = 1, \qquad \sigma = 1 \text{ (log utility)}, \qquad \beta = 0.01429. \tag{45}$$

Our goal is to perform comparative dynamics analysis under ten different values of the policy parameter, $0 \leq \tau < 1$, which are assumed to be constant over time. Specifically, we consider

$$\tau = 0, 0.1, 0.2, \ldots, 0.9. \tag{46}$$

It is worth to remind that in terms of rent appropriability by *R&D*-firms via IPR, $\tau = 0$ supplies to the right holder the maximal degree, while the other values indicate progressively decreasing appropriability levels, reaching a minimum when $\tau = 0.9$.

Recall from Proposition 4 (i) that to each τ-value in (46) corresponds a different (transitory) turnpike $\tilde{k}_\tau(A)$ as defined in (38), each lying one below the other for increasing values of parameter τ, with $\tilde{k}_0(A)$ – *i.e.*, the turnpike corresponding to $\tau = 0$ – on top, characterizing a scenario in which the largest capital/knowledge ratio is optimal to sustain growth, and $\tilde{k}_{0.9}(A)$ at the bottom, envisaging an equilibrium with smaller capital/knowledge ratio and closer to the the first-best solution. Each curve converges to its own linear asymptotic turnpike, $\tilde{k}_\tau^\infty(A)$, as defined in (39), which are parallel lines corresponding to long-run balanced growth with the same constant growth rate given by (43):

$$\gamma = 0.0198. \tag{47}$$

[21]The β value has been chosen in order to contain the error in the simulation of the optimal policy in all three tax-subsidy scenarios considered in (46). Note that the parameter values we have chosen imply a growth rate of around 2% in (47), which is very close to that in Acemoglu and Akcigit (2012).

Figure 1 depicts in the (A, k) space the stagnation line, $\hat{k}(A) = (156.25) A$, in dark grey and three turnpike curves in *light grey*, *dark grey* and *black*, corresponding to $\tau = 0$, $\tau = 0.5$ and $\tau = 0.9$ respectively. We shall identify with these three colors all relevant curves related to these three examples in subsequent figures throughout the paper. Consistently with results in Privileggi and Marsiglio (2014), the graph of each turnpike, $\tilde{k}_\tau(A)$, is a U-shaped curve; indeed, transition dynamics along the turnpike foresee a decreasing capital $\tilde{k}_\tau(t)$ when t is small, which becomes increasing for larger t. We set[22] $A_0 = 2.2190$ as the initial stock of knowledge, common to all three tax-subsidy regimes, to which correspond three initial capital endowments on each turnpike: $\tilde{k}_0(A_0) > \tilde{k}_{0.5}(A_0) > \tilde{k}_{0.9}(A_0)$.

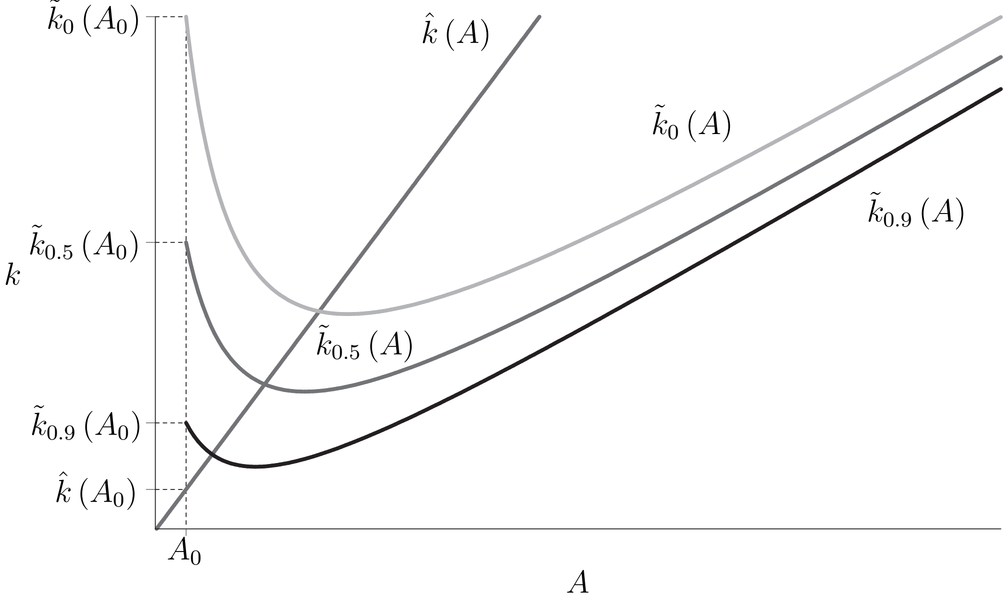

FIGURE 1: the stagnation line and three turnpikes, for $\tau = 0, 0.5, 0.9$.

Preliminary simulation results pertain to the Skiba-type conditions needed for starting the growth process and are listed in Table 1: it is remarkable the decreasing pattern of both the initial stock of knowledge, $\tilde{A}_\tau^{sk}$, and stock of capital, $\tilde{k}_\tau^{sk}$, required to allow the economy to take off toward sustained growth when starting on the turnpike, with respect to increasing τ-values. In other words, larger values of parameter τ relieve the initial conditions required for the economy to grow in the long run, allowing poorer countries to undertake a growth path. It is also noticeable from the second column in Table 1 that, as $\tilde{A}_\tau^{sk} > A_0 = 2.2190$ for all $\tau = 0, \ldots, 0.9$, if the economy were supposed to start with an initial capital endowment corresponding to $\tilde{k}_\tau(A_0)$ on each turnpike, none of the ten scenarios considered in our study would allow a sustained growth pattern under our choice of initial stock of knowledge $A_0 = 2.2190$, as the social planner would find optimal to lead the economy to eventual stagnation, regardless the tax-subsidy policy chosen.

In order to perform a truly comparative dynamics analysis across all different values of the tax-policy parameter τ listed in (46), we must, however, fix a unique starting point common to all tax-subsidy scenarios, which implies that at least nine over ten cases considered in (46) envisage an early finite period of transition outside the turnpike. As observed in Privileggi (2013), trajectories starting from initial conditions (A_0, k_0) that lie 'above' the turnpike at $t = 0$ are easier to handle from a numerical point of view; therefore, we choose point $(A_0, k_0) =$

[22]See Privileggi and Marsiglio (2014) for a technical discussion on such a choice for the initial value of A_0.

τ	$\tilde{A}_{\tau}^{sk}$	$\tilde{k}_{\tau}^{sk}$
0	3.0149	581.4472
0.1	2.9629	563.8338
0.2	2.9075	545.8273
0.3	2.8486	527.3486
0.4	2.7851	508.3637
0.5	2.7154	488.8517
0.6	2.6400	468.4889
0.7	2.5572	447.0335
0.8	2.4603	424.6582
0.9	2.3462	400.5483

TABLE 1: Skiba-points in terms of initial knowledge, $\tilde{A}_{\tau}^{sk}$, and capital, $\tilde{k}_{\tau}^{sk}$, on each turnpike.

$\left(A_0, \tilde{k}_0 (A_0)\right) = (2.2190, 887.6010)$, where $\tilde{k}_0 (A_0) = 887.6010$ is the initial capital associated to the (common) initial knowledge $A_0 = 2.2190$ on the 'highest' turnpike among the three, corresponding to the lowest tax-subsidy scenario ($\tau = 0$ under full decentralization). This is the left endpoint on the light grey curve on the top left corner of Figure 1. Note that our choice for the A_0-value is such that the corresponding initial capital values on each turnpike lie all above the capital value $\hat{k} (A_0) = 346.7191$ on the stagnation line (the point on the bottom left corner in Figure 1), for all τ-values in (46); that is, $\tilde{k}_{\tau} (A_0) > \hat{k} (A_0)$ for all $\tau = 0, \ldots, 0.9$; moreover, by construction such a point is also strictly above the other two turnpikes – in dark grey and black respectively – in Figure 1.

For all but the first τ-value, *i.e.*, for $\tau = 0.1, \ldots, 0.9$, we must thus consider an early period of time, from $t = 0$ to the instant $t_0 > 0$, corresponding to a knowledge stock value that we label A_{τ}^{r}, during which the optimal time-path starts from the initial condition (A_0, k_0) – which lies strictly above all values $\tilde{k}_{\tau} (A_0)$ for $\tau > 0$ – before entering their own turnpike at $t = t_0$ and keep growing along it thereafter. Any such type of optimal trajectory must satisfy the last necessary condition in (26), $\delta / \phi_{\tau} (A) > \lambda$, associated to the largest possible investment in R&D activities by the government:[23] $G = y = \theta k^{\alpha} A^{1-\alpha}$. In other words, from the social planner point of view it is optimal to invest all the output into the production of knew knowledge along such early-transition trajectories, during which knowledge is being piled up until the level A_{τ}^{r} is reached.

Figure 2 plots the whole time-path trajectories for $\tau = 0$, $\tau = 0.5$ and $\tau = 0.9$. The light grey curves, corresponding to the no-tax decentralized regime $\tau = 0$, denote trajectories starting already on the $\tilde{k}_0 (A)$ turnpike at $t = 0$ which keep moving along it thereafter, while the dark grey and black curves correspond to the active tax-subsidy regimes $\tau = 0.5$ and $\tau = 0.9$. According to the method provided by Privileggi and Marsiglio (2014), trajectories of the latter type are built as *piecewise functions* by joining each trajectory above the turnpike (for $0 \leq t \leq t_0$) with its 'continuation' along the turnpike (for $t \geq t_0$) at the instant[24] t_0, corresponding to the knowledge value A_{τ}^{r} at which each turnpike is hit. Because $G(t) = y(t) = \theta [k(t)]^{\alpha} [A(t)]^{1-\alpha}$ for $0 \leq t < t_0$ while $G(t) = \tilde{G}(t) < y(t)$ for $t \geq t_0$, where $\tilde{G}(t)$ is given by (41), a discontinuity 'jump' for the control G occurs at instant $t = t_0$, while all other trajectories exhibit a kink

[23]See Proposition 1 on p. 3464 in Tsur and Zemel (2007).

[24]From the third column in Table 2 we see that all intersection points are reached at instants t_0 that are very close to each other in all tax-subsidy regimes, ranging from $t_0 = 8.30$ when $\tau = 0.9$ to $t_0 = 8.80$ when $\tau = 0.3$ and $\tau = 0.4$.

on $t = t_0$, where they are not differentiable. Such policy, even if it is being pursued only for an early finite period of time, turns out to be substantially beneficial in that it translates into steeper $A(t)$, $y(t)$ trajectories for $0 \leq t \leq t_0$ [dark grey and black curves in Figures 2(a) and 2(c)], indicating a high boost to growth provided during the initial paths covered above the turnpikes. This initial boost, in turn, explains the better performances of all $A(t)$, $k(t)$, $y(t)$ and $c(t)$ trajectories under the $\tau > 0$ regimes (*e.g.*, those in dark grey and black in Figure 2) than their counterparts for $\tau = 0$ (in light grey in Figure 2) when t becomes large.

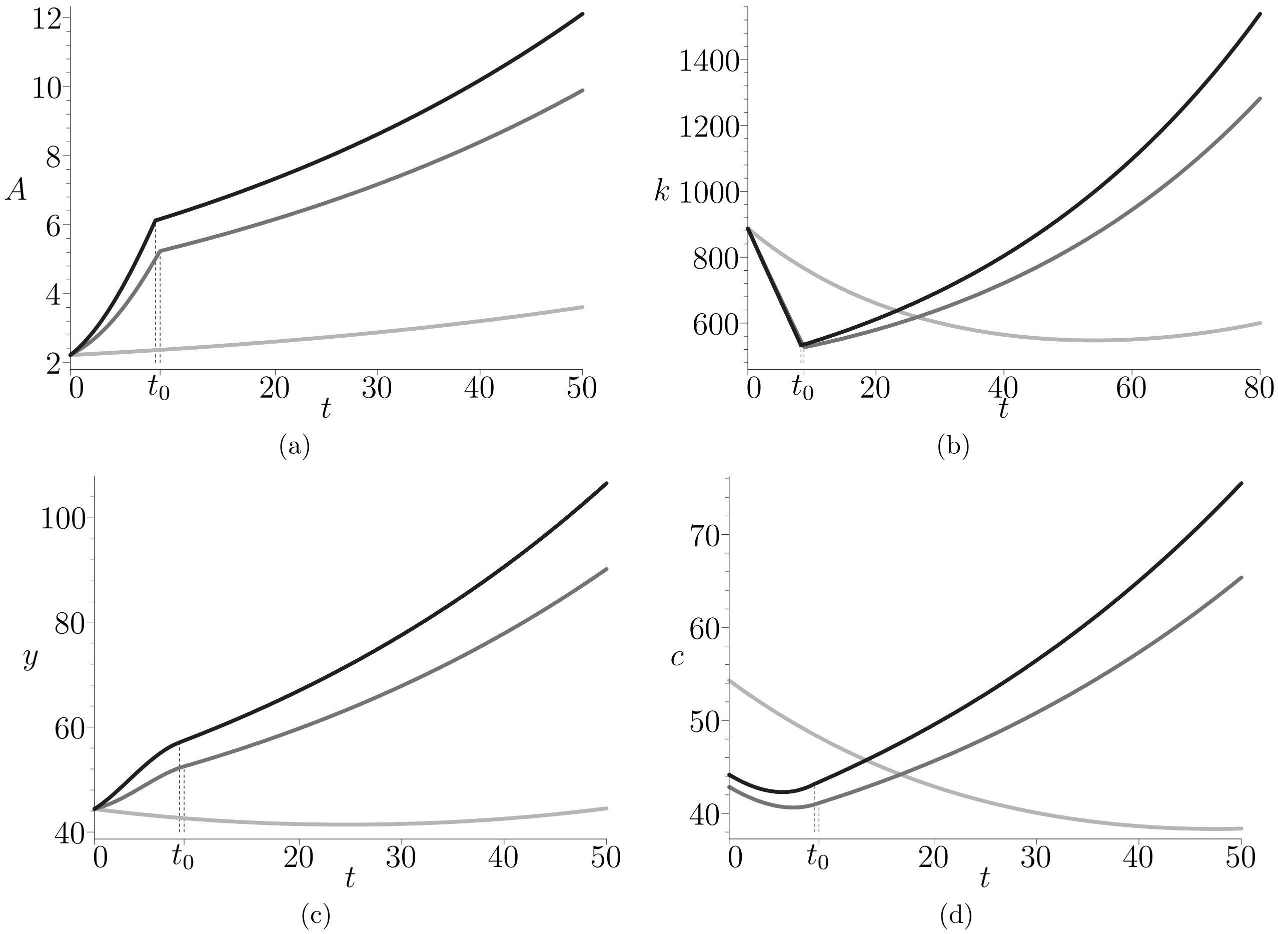

FIGURE 2: whole optimal transition time-path trajectories, (a) $A(t)$, (b) $k(t)$, (c) $y(t)$ and (d) $c(t)$, under the three tax-subsidy regimes, $\tau = 0$ (full decentralization) in light grey, $\tau = 0.5$ in dark grey and $\tau = 0.9$ in black, all starting from $A_0 = 2.2190$ and $k_0 = 887.6010$ in $t = 0$.

From figure 2(d), however, it is not clear what the effect of the different policies on welfare may be, as in early times the $\tau = 0.5$ and $\tau = 0.9$ regimes envisage a lower consumption than in the $\tau = 0$ regime, only to catch up and rapidly overcome later on. As a matter of fact, the former trajectories may fail to deliver a welfare higher than that yield by the latter if discounting assigns less weight to later consumption than that assigned to early consumption.

The last two columns of Table 2 report our main welfare results.[25] They clearly indicate that

[25]From Table 1 we know that, when $\tau = 0$ and $(A_0, k_0) = (2.2190, 887.6010)$ the economy does not satisfy the Skiba condition, which yields the first value in the fourth column of Table 2, and implies that the first number in the fifth column comes from the welfare value estimated for the economy ending up in stagnation. All other numbers in the fifth column are larger than the latter; hence, for all values $\tau > 0$, (A_0, k_0) satisfies the Skiba condition, yielding the last nine values in the fourth column of Table 2.

the initial push deriving from starting above any turnpike, if on one hand entails sacrificing consumption in early times as shown by Figure 2(d), on the other hand it turns out to be sufficiently strong to 1) let the economy escape the stagnation trap otherwise forecasted by Table 1 for all active-policy regimes in which $\tau > 0$, and 2) yield a social welfare that is strictly increasing in the τ parameter values, as it is apparent from the fifth column. Indeed, consistently with the first row in Table 1, when $\tau = 0$ our economy happens to be born on the highest turnpike $\tilde{k}_0(A_0)$ and doomed to renounce growth in the long-run as the Skiba condition is not satisfied. Conversely, whenever an active tax-subsidy policy, even a mild one (already a small but strictly positive tax-subsidy value as $\tau = 0.1$ makes the difference), is being implemented by the authorities, the same economy is capable to take off toward long-run sustained growth, as confirmed by all rows except the first one in the fourth column of Table 2.

τ	A^r_τ	t_0	Toward	Social welfare
0	2.2190 ($= A_0$)	0	stagnation	95.8745
0.1	4.3638	8.64	ABGP	96.1080
0.2	4.5887	8.76	ABGP	96.5087
0.3	4.8050	8.80	ABGP	96.9256
0.4	5.0194	8.80	ABGP	97.3601
0.5	5.2342	8.76	ABGP	97.8152
0.6	5.4514	8.68	ABGP	98.2929
0.7	5.6711	8.58	ABGP	98.7914
0.8	5.8921	8.45	ABGP	99.3064
0.9	6.1187	8.30	ABGP	99.8471

Table 2: Intersection points A^r_τ between each trajectory from above and its turnpike, instants t_0 at which the intersection occurs, type of equilibrium (toward either stagnation or sustained growth along an ABGP) and social welfare in all tax-subsidy regimes.

To conclude, the approximated values found through the whole procedure discussed so far lead to the following results:

i) under *no tax-subsidy scheme* ($\tau = 0$), *i.e.*, 'full' private IPR rent appropriability by *R&D*-firms ($1 - \tau = 1$), *the economy ends up in stagnation*, while

ii) already under a *'mild' active tax-subsidy scheme* ($\tau \geq 0.1$), *i.e.*, 'mild' IPR appropriability ($1 - \tau \leq 0.9$), *the economy grows along an ABGP in the long-run*,

iii) *social welfare is strictly increasing with respect to positive increments of the tax-subsidy parameter* τ – corresponding to progressively *'stronger' tax-subsidy schemes*, *i.e.*, progressively 'weaker' appropriability.

This result is somewhat surprising, since it implies that capital crowding out has positive effects on welfare. However, this is explained by the power of recombinant technological progress, which at early times makes it convenient to take away some capital from the output producing sector to be used for consumption, thus freeing resources coming from the current output to invest them into knowledge advances.

6 Discussion

Our findings highlight the importance of strategic choices in the early stages of development, in which a large reallocation of resources between economic sectors may be actually needed in order to avoid stagnation and start a growing process. As far as developed countries are concerned, tax-subsidies schemes can support a lighter and more dematerialized economy, in which the capital to knowledge ratio is shifted from physical capital to knowledge.

While so far the literature has largely recognized that government intervention is needed to fix the inefficiencies deriving from the IPR system, it has mainly focused on the appropriability of knowledge considering the rights' structural features (*e.g.*, patent breadth or length; see Acemoglu and Akcigit, 2012, and the literature quoted therein). We contend here that the focus should be shifted to the rents to which the IPR system gives rise, and to the corrective role that taxation can play in improving social welfare. Indeed, taxes can be efficiently levied when rents are at stake: in the case here investigated this solution is aimed at limiting the appropriability of rents granted by IPR. Governments precisely tax profits deriving from IPR, and patents – but also trademarks and copyrights – are becoming a more and more fundamental component of assets of corporations. Taxation in this field, however, is often avoided by multinational corporations through the resort to various techniques. The registration of patents can, *e.g.*, be made by a subsidiary operating in a country applying low tax rates. This affiliate then collects profits from royalties, often paid by other firms belonging to the same corporation which apply inflated transfer prices in order to shift profits to the low tax country (Karkinsky and Riedel, 2012). Governments can resort to taxation schemes that close such loopholes, through stricter rules about the imputation of profits at the corporation's residence country.[26] The recent public economics literature depicts taxation of capital income (capital in this framework, unlike in our model, includes IPR) in a more favorable way than in the past (see, *e.g.*, Diamond and Saez, 2011, or, for a dynamic setting, Aghion et al., 2013), as part of a strategy for fighting also other forms of tax avoidance and for improving equity and efficiency.

Among the features that allow the simplified patent system sketched out in this paper to ensure in the long run a convergence to the same growth pattern as that characterizing a fully efficient system, there is the fact that the R&D outputs bought by the government are immediately made available both to the production sector and to the other researchers, so that the process of knowledge recombination can go on smoothly. The real world differs from the model in the length of patent protection as it implies in many jurisdictions a delay up to 20 years, which thus more severely affects the recombinant process than in our stylized framework. Like in the model, instead, governments are boosting to some extent the demand for R&D through various forms of financial support. Nonetheless, and somewhat surprisingly, they have by and large embraced an approach that allows researchers in public universities and laboratories to patent their results, thus contradicting the quest for a quick and easy dissemination of new knowledge.

[26]The CFC (Controlled Foreign Company) rule makes patent income earned by subsidiaries in low tax countries taxable at the parent location. It aims at preventing erosion of the home country tax base, by denying the deferral of profit taxation until repatriation. The conditions for rendering the income immediately taxable for the parent company vary from country to country, and usually refer to the characteristics of the country in which the controlled company is located (which can, *e.g.*, be included in a black or grey list of likely tax heavens) or to the type of income. Royalties from patents are often considered taxable as long as they represent passive income, *i.e.*, they just generate a rent and are not directly used for productive purposes by the controlled firm. The European Commission is preparing an action plan on these topics to prevent tax erosion. The topic is highly debated particularly in relationship to the concentration of the Google profits earned in Europe into the Irish subsidiary.

7 Concluding Remarks

Since the early studies on economic growth, none doubts on the crucial role of technological progress and knowledge accumulation on determining the long-run economic performance. Debated and delicate is instead how knowledge creation should be fostered and what are the consequences of the IPR system. In fact, part of the growth and innovation literature asserts that properly designed economic incentives relying upon the private appropriability of knowledge are the key element for individual agents to invest in R&D activities. Historians however contend that many other aspects and different incentive systems may be equally appealing for individual agents, and able to promote knowledge advances. In order to combine these different views, we construct an endogenous growth model where the role of markets and independent knowledge production are interrelated.

We introduce a market mechanism as part of the recombinant knowledge creation, originally proposed by Weitzman (1998). The government plays a crucial role in our model: it collects taxes from the final producing sector to finance R&D activities, and relies on a tax-subsidy scheme in order to correct for the inefficiencies generated by the private appropriation process. We show that government activity may be needed in order to avoid stagnation and drive the economy along a growth path. While in the very long run, the economy converges towards an (asymptotic) balanced growth path, along which the economic growth rate is independent of the policy parameter, the IPR system and the appropriability policy adopted jointly affect the transitional dynamics and therefore social welfare.

Because transitional dynamics in recombinant growth models are tough objects, we rely on quantitative methods to perform comparative dynamic analysis. Following Privileggi and Marsiglio (2014) computations, we show that *welfare increases in the tax-subsidy* scheme, that is, in the bound on the appropriability via IPR: larger values of τ yield larger welfare in the economy. Moreover, in the example studied in Section 5 we have seen that a *no tax-subsidy scheme* ($\tau = 0$), *i.e.*, full appropriability of the IPR rent, actually leads the economy toward stagnation while public intervention may boost sustained growth.

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