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## THE IMPACT OF TAX REFORMS ON THE LABOR SUPPLY OF MARRIED WOMEN

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# **The impact of tax reforms on the labor supply of married women**

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## **Abstract**

We show how a neoclassical labor supply model with optimal decisions for labor force participation and hours of work, derived from first order conditions, can be taken to data even in the presence of a step-wise linear progressive tax system which may imply non-convex budget sets. The estimated model is used to simulate the optimal behavior when the tax system of 2001 is replaced by the less progressive tax system of 2006. The latter tax system implies a lower labor market participation among married women in Norway, a higher working load, given participation, and a more uneven distribution of household income.

**Keywords:** Labor supply, non-convex budget sets, marginal criteria

**JEL classification:** J22, C51

## **1. Introduction**

Two rather different approaches have been applied to estimate the neoclassical labor supply model. In Dagsvik et al (1995) and van Soest (1995) hours are made discrete and the utility function includes a stochastic taste shifter. The random component is assumed to be iid extreme value. These discrete choice models can readily handle the non-convexity of the budget sets. The distribution

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of the optimal choice of hours, including zero hours, is derived from comparisons of utility levels, given the budget sets.

The second approach, sometimes referred to as the Hausman approach and initiated by Burtless and Hausman (1978) and Hausman (1979), accounts for the entire budget set being generated by a nonlinear tax system, which could imply a non-convex budget set. In order to ensure convex preferences at kink points of the budget set the Hausman approach requires additional ad hoc restrictions on the model beyond those that follow from economic theory, (MaCurdy et al. (1990), MaCurdy (1992) and Heim and Meyer (2003)). In particular, the supply of hours is assumed to be a linear function of the marginal wage rate and virtual income along the step-wise segments of the budget constraint.

In this paper we demonstrate how the neoclassical labor supply model can be estimated when labor market participation and continuous hours follow from first order conditions. Like in the Hausman approach, all aspects of the budget set is accounted for, including the non-convexity of the budget set. Unlike the Hausman approach, we do not assume linear supply functions.

In Section 2 we outline the neoclassical labor supply model when the budget set is non-convex. In Section 3 we propose a full maximum likelihood approach to estimate the model, which allows to estimate jointly the wage equation, labor market participation and continuous hours in one step. This approach is computationally more convenient than the classical Heckman step-wise procedure and yields comparable but more precise estimates. Sections 4 and 5 describe our unique dataset and present the estimates from both the traditional stepwise procedure and our one-step method. Section 6 discusses the results of tax and wage increase simulation and computes the extensive and intensive elasticities of labor supply. Section 7 concludes.

## 2. The classical labor supply model with non-convex budget sets

Let  $U(C,L)$  be the utility function of an individual. It is assumed to depend on annual consumption  $C$  (equal to disposable income) and leisure  $L$ . We will assume that the utility function is additive separable in the utility of consumption and leisure. Moreover, let  $W$  denote hourly wage,  $h$  annual hours of work,  $I$  non-labor income and  $T$  the tax function. In the classical text-book approach to labor supply, the individual is assumed to maximize utility, given the budget set:

$$\text{Max}U(C,L)$$

subject to

$$(1) \quad C \leq Wh + I - T(Wh, I)$$

$$(2) \quad L = I - \frac{h}{M}$$

$$(3) \quad h \geq 0$$

where  $M=3650$  is the total number of hours a year available for work; the remaining hours is needed for rest and sleep.

Let  $R(W, h, I)$  denote the marginal rate of substitution between leisure and consumption, also called the *shadow price* of leisure, that is

$$(4) \quad R(W, h, I) = \frac{\partial_2 U(C, L)}{\partial_1 U(C, L)}$$

where  $\partial_j$  denotes the partial derivative with respect to component no  $j$ .

Let  $m(W, h, I)$  denote the marginal wage rate, given by

$$(5) \quad m(W, h, I) = W(1 - \partial_1 T(Wh, I))$$

According to the neoclassical labor supply model the individual will participate in the labor market if the reservation wage is less than or equal to the marginal wage rate, evaluated at zero hours.

$$(6) \quad R(W, 0, I) \leq m(W, 0, I)$$

Given participation, hours supplied is given by equating the shadow price of leisure and the marginal wage rate:

$$(7) \quad R(W, h, I) = m(W, h, I)$$

where  $h$  denotes optimal hours.

The main problem with taking this neoclassical labor supply to data is that in most countries the marginal tax rate is not uniformly increasing with income. This implies that the budget set is non-convex. Moreover the tax function is in most cases a step-wise linear function of taxable income. Applying the first order condition (7) requires that the tax function is smoothed. A more serious problem is that there could be local and global optima and one needs an econometric specification that distinguishes between these possible optima.

In our approach here (continuous) hours supplied is given implicitly by Equation (7) and participation by Equation (6). Unlike the Hausman-approach we will not assume a linear labor supply function. Instead we assume a functional form for the utility function that is rather flexible and can be given an axiomatic justification, see Dagsvik and Strøm (2006). The utility function is assumed to be a Box-Cox transformation of consumption and leisure. Attached to the leisure term we have added a stochastic taste shifter. The tax function is based on a smoothed version of the step-wise linear tax function, which is given in Appendix A. The smoothing implies that the kinks are removed. The smoothed tax function still implies that the marginal tax rates are not uniformly increasing with income and hence the non-convexity of the budget set is preserved. The smoothed tax function and how well it fits with the tax rules is given in Appendix A.

Inserting from (1) and (2) we observe that utility  $U$  is a function of annual hours. Our specification of the utility function is the following:

$$(8) \quad U(h) = U_1(h) + e^{\varepsilon} U_2(h)$$

where

$$(9) \quad U_1(h) = \frac{(C(h) - C_0)^{\alpha_1} - 1}{\alpha_1}$$

$$(10) \quad U_2(h) = e^{X_2 b} M \frac{(1 - \frac{h}{M})^{\alpha_2} - 1}{\alpha_2}$$

$C_0$  is minimal consumption necessary and  $X_2$  is a vector of individual characteristics. If  $(\alpha_1, \alpha_2)$  both are less than one, the utility function is strictly concave. When  $(\alpha_1, \alpha_2)$  approaches zero, the utility function becomes log-linear in the two arguments.  $\varepsilon_2$  is a stochastic taste shifter.

Let  $h$  denote the optimal hours of work that we observe that the individual has chosen. In our model it follows from the first order condition:  $\frac{\partial U(h)}{\partial h} = 0$ . To this end we call this approach to labor supply the marginal criteria approach.

Like we mentioned above the non-convexity of the budget set may imply that there are more than one tangency point between the budget curve and the indifference curves. Suppose that there are three (more than three is a trivial extension), and the associated hours are  $(h, h_1, h_2)$ . If  $h$  yields the global optima, then the following must hold:

$$(11) \quad U_1(h) + e^{\varepsilon_2} U_2(h) \geq U_1(h_1) + e^{\varepsilon_2} U_2(h_1)$$

$$(12) \quad U_1(h) + e^{\varepsilon_2} U_2(h) \geq U_1(h_2) + e^{\varepsilon_2} U_2(h_2)$$

or

$$(13) \quad \varepsilon_2 \leq \log \left[ \frac{U_1(h) - U_1(h_1)}{U_2(h_1) U_2(h)} \right] \equiv G(h, h_1)$$

$$(14) \quad \varepsilon_2 \leq \log \left[ \frac{U_1(h) - U_1(h_2)}{U_2(h_2) U_2(h)} \right] \equiv G(h, h_2)$$

This can be re-written as:

$$(15) \quad \varepsilon_2 \leq \min[G(h, h_1), G(h, h_2)]$$

This makes it possible to employ the marginal criteria approach in the presence of budget sets that are generated by non-linear tax rules and where the

budget sets are even non-convex. We observe that the right hand side of (15) is deterministic. Applying the marginal criteria,  $h$  is given by:

$$(16) \quad \frac{\partial U_1(h)}{\partial h} + e^{\varepsilon_2} \frac{\partial U_2(h)}{\partial h} \geq 0$$

Concentrating for the moment on strictly positive hours, we observe that from Equation (16) we get

$$(17) \quad \varepsilon_2 = \log \frac{\partial U_1(h)}{\partial h} - \log \left( -\frac{\partial U_2(h)}{\partial h} \right) \equiv g(h)$$

Let  $f(\varepsilon_2)$  be the density function in the distribution of  $\varepsilon_2$ . Then we get the following probability for the optimal hours of work:

$$(18) \quad P(h \in (x, x + dx)) = f(g(x)) \frac{dg(x)}{dx} 1\{g(x) \leq \min(G(x, h_1), G(x, h_2))\} dx$$

The indicator function  $1\{g(x) \leq \min(G(x, h_1), G(x, h_2))\}$  equals one if the optimal hours of work is chosen, and zero otherwise, and hence it can be ignored in the likelihood employed in estimating the model.

It should be noted that once the model is estimated we can use the utility function directly when assessing different policy issues like tax reforms. This means that we do not need to employ the rather cumbersome first order conditions to assess the reforms. The assessment can be done in terms of labor supply responses. In Section 6 we give an example where the tax system in 2001 is compared to the tax system of 2006. The tax reform in 2006 reduced the marginal tax rates at the top rather considerably.

### 3. The econometric model

In order to estimate the labor supply model, where all individuals have the option of not working, we need to specify a wage equation. We assume that wages are log-normally distributed:

$$(19) \quad \log W = X_i \gamma + \varepsilon_i,$$

where  $X_i$  is a vector of individual characteristics.



The random term in the wage equation,  $\varepsilon_1$ , and the random taste shifter,  $\varepsilon_2$ , are assumed to be jointly normally distributed with zero mean. Because of this assumption we can write:

$$(20) \quad \varepsilon_2 - \varepsilon_1 = \theta\varepsilon_1 + \varepsilon_3$$

where  $\varepsilon_3$  is a zero mean normal that is independent of  $\varepsilon_1$  and  $\theta$  is a constant. The covariance between the error term in the wage equation and the taste shifter is then  $cov(\varepsilon_1, \varepsilon_2) = (1 + \theta)\sigma_1^2$ , where  $\sigma_1^2$  is the variance in the wage distribution.

From Equations (17) and (19) we get that optimal hours of work, given participation, is determined by:

$$(21) \quad \varepsilon_2 - \varepsilon_1 = (1 - \alpha_2) \log\left(1 - \frac{h}{M}\right) + (\alpha_1 - 1) \log(C(h) - C_0) + \log(1 - \partial_1 T(Wh, I)) + X_1\gamma - X_2b$$

Applying (20) and substituting for  $\varepsilon_1 = \log W - X_1\gamma$ , Equation (21) can be rewritten to yield:

$$(22) \quad \varepsilon_3 = -\theta \log W + (1 - \alpha_2) \log\left(1 - \frac{h}{M}\right) + (\alpha_1 - 1) \log(C(h) - C_0) + \log(1 - \partial_1 T(Wh, I)) + (1 + \theta)X_1\gamma - X_2b$$

In order to specify the likelihood for those individuals that are observed working we need to find the distribution for the observable random variables,  $h$  and  $W$ , given the assumed distribution of the random variables  $\varepsilon_3$  which appears in (22) and  $\varepsilon_1$  in Equation (19). The relevant Jacobian for this problem is the following:

$$(23) \quad \det \begin{vmatrix} \frac{\partial \varepsilon_1}{\partial h} & \frac{\partial \varepsilon_1}{\partial W} \\ \frac{\partial \varepsilon_3}{\partial h} & \frac{\partial \varepsilon_3}{\partial W} \end{vmatrix} = \frac{1}{W} \left| \frac{\partial \varepsilon_3}{\partial h} \right|$$

where

$$(24) \quad \frac{\partial \varepsilon_3}{\partial h} = \left[ \frac{(\alpha_1 - 1)W(1 - \partial_1 T(Wh, I))}{C(h)} + \frac{(\alpha_1 - 1)}{M - h} - \frac{W \partial_1^2 T(Wh, I)}{1 - \partial_1 T(Wh, I)} \right]$$

It is interesting to note that

$$(24) \quad \frac{\partial \varepsilon_3}{\partial h} = - \left[ \frac{1}{h E_s(W, h)} + \frac{W \partial_1^2 T(W, h, I)}{1 - \partial_1 T(W, h, I)} \right],$$

where  $E_s(W, h)$  is the Slutsky elasticity. We observe that the second derivative of the tax function is present in the Jacobian determinant.

Let  $S_2$  be the subsample of individuals that work. The likelihood for the subsample of these individuals are

$$(25) \quad L_2 = \prod_{i \in S_2} \Phi' \left( \frac{\log(1 - \partial_1 T(W_i h_i, I_i)) + (\alpha_1 - 1) \log C(W_i h_i, I_i) - \theta \log W_i + X_{1i} \gamma (\theta + 1) - X_{2i} b + (1 - \alpha_2) \log \left(1 - \frac{h}{hM}\right)}{\sigma_3} \right) \times \left| \frac{\partial \varepsilon_{3i}}{\partial h_i} \right| \frac{1}{W_i} \Phi' \left( \frac{\log W_i - X_{1i} \gamma}{\sigma_1} \right) \frac{1}{\sigma_1}$$

where  $\Phi'(\cdot)$  is the standard normal density function and  $\sigma_1^2 = \text{Var} \varepsilon_{1i}$  and  $\sigma_3^2 = \text{Var} \varepsilon_{3i}$ .

The condition for non-participation that follows from our specification of the classical labor supply model is given by:

$$(26) \quad (\alpha_1 - 1) \log(C(0) - C_0) + X_{1i} \gamma - X_{2i} b + \log(1 - \partial_1 T(0, I)) \leq \varepsilon_2 - \varepsilon_1.$$

Note that from Equation (20) we have  $\text{var}(\varepsilon_2 - \varepsilon_1) = \theta^2 \sigma_1^2 + \sigma_3^2$ . Moreover,  $\partial_1 T(0, I) = 0$

Let  $S_1$  be the subsample of individuals that do not work. The likelihood for non-working individuals equals

$$(27) \quad L_1 = \prod_{i \in S_1} 1 - \Phi \left( \frac{(\alpha_1 - 1) \log C(0, I_i) + X_{1i} \gamma - X_{2i} b}{\sqrt{\theta^2 \sigma_1^2 + \sigma_3^2}} \right) = \prod_{i \in S_1} \Phi \left( \frac{(1 - \alpha_1) \log C(0, I_i) - X_{1i} \gamma + X_{2i} b}{\sqrt{\theta^2 \sigma_1^2 + \sigma_3^2}} \right)$$

The parameters to be estimated are  $\alpha_1, \alpha_2, b, \gamma, \theta, \sigma_1, \sigma_2$ . They are determined by maximizing the total log-likelihood,  $\log L$ , that is  $\log L = \log L_1 + \log L_2$ .

In Appendix B we show how the model can be estimated in a step-wise (Heckman) procedure.

#### **4. Data**

We use data from the 2001 wave of the Annual Labor Force Survey by Statistics Norway. The survey data is in line with the definitions and guidelines of the International Labour Organization (ILO) and EU/Eurostat. The main advantage of this dataset, beginning in 2001, is that the employment data is based on administrative registers and as such is pretty detailed. Working days lost due to labor conflicts are compiled by Statistics Norway on the basis of information supplied mostly by the labor and employer's organizations.

For our analysis we focus on the subsample of married women aged 26-59. Employment is defined as working for pay or profit for at least one hour in the reference week, or who were temporarily absent from work because of illness or holidays. The number of hours of work is computed as to include all actual working hours, including overtime and excluding absence from work. Overtime is defined as working hours which exceed the settled or contractual working hours for full-time employees, conducted during a specified reference week. For individuals who are employed at several jobs, the total number of hours is defined based on the primary and secondary jobs.

Marital status is self-reported, and we recode cohabitants as married. Non currently married include never married and previously married, which in turn includes widow, widowers, separated and divorced individuals. Table 1 present the summary statistics for the sample.

When estimating the model, some outliers are dropped. In particular, we drop all households in the 1st and 10th deciles of disposable income and those with missing information. In estimating the model we have used the tax rules of 2001, given in Appendix A, and smooth is out as given in Appendix A.

Furthermore, we fix the minimum consumption level,  $C_0$ , to NOK 51,360 for 2001 and NOK 54,955 for 2006. AS of April 2015 1USD is approximately equal to NOK 8.

**Table 1. Summary statistics, married women, Norway 2001.**

<b>Variable</b>	<b>Mean</b>	<b>Std.dev</b>	<b>Min</b>	<b>Max</b>
Weekly hours	30.62	10.92	0	74
Annual hours	1,408.63	502.63	0	3,404
Working Status <sup>5</sup>	1.04	0.29	1	3
Hourly wage, NOK	166.31	67.36	0	595
Non-labor income, NOK	29,949.10	49,864.12	2.88	380,807
Married 2, cohabiting 3.	2.04	0.20	2	3
Age	44.40	8.72	26	59
Years of Education	13.64	2.66	7	20
No of children, 0-2 years	0.15	0.40	0	2
No of children, 3-6 years	0.11	0.32	0	2
No of children, 7-17years	0.60	0.85	0	5

## 5. Estimates

The estimates of the joint model are given in Table 2 and the estimates of the stepwise model in Table 3. The estimates are fairly similar, with the exception that the exponent related to leisure in the Box-Cox utility function is slightly higher in the step-wise estimation. To this end we proceed with the estimates of the joint model.

The estimates of the exponents related to consumption and leisure are both significantly less than 1. Thus the utility function is strictly concave. Children between 0 and 2 and between 7 and 17 have a positive and significant impact on the utility of leisure, suggesting that married women with young children or children above 7 have a weaker incentive to participate in the labour market and/or to work longer hours than say, women with children between 3 and 6. While this might seem odd at first, one likely explanation is that childcare

<sup>5</sup> 1= employed, 2=unemployed, 3= out of the labor force (OLF)

facilities for children between 3 and 6 are available for almost all families in Norway.

From the estimates we observe that the correlation between the error terms in preferences and wages is significant and positive, and quite high. The estimates imply that

$$\text{var}(\varepsilon_2) = (1 + \theta)^2 \sigma_1^2 + \sigma_3^2 = 0.1580, \text{ and } \text{corr}(\varepsilon_1, \varepsilon_2) = \frac{\text{cov}(\varepsilon_1, \varepsilon_2)}{\sigma_1 \sigma_2} = 0.8825$$

**Table 2. Maximum likelihood estimates of the joint model**

<b>Variables</b>	<b>Coefficients</b>	<b>Estimates</b>	<b>t-values</b>
<b>Exponents:</b>			
Consumption	$\alpha_1$	0.9535	89.9
Leisure	$\alpha_2$	0.5982	15.6
<b>Leisure:</b>			
Constant	$b_0$	-2.8198	-17.1
Age/10	$b_1$	-0.0213	-0.3
(Age/10) <sup>2</sup>	$b_2$	0.0081	1.0
Children 0-2	$b_3$	0.0567	4.7
Children 3-6	$b_4$	0.0194	1.5
Children 7-17	$b_5$	0.0320	5.7
<b>Wage equation:</b>			
Constant	$\gamma_0$	-2.3929	-48.8
Years of educ./10	$\gamma_1$	0.2849	15.7
Experience/10	$\gamma_2$	0.1028	3.7
(Experience/10) <sup>2</sup>	$\gamma_3$	-0.0157	-2.9
Standard deviation	$\sigma_1$	0.3365	68.1
Correlation	$\theta$	0.0425	2.0
Standard deviation	$\sigma_3$	0.1869	24.5
No of observations		2,404	
Log-likelihood		-9,394.03	

In the step-wise procedure, we estimate first the reduced form participation equation,  $\Phi(Z\beta)$ , where  $Z$  contains a constant, age in linear and squared form, and net non-labor income. The estimated selection term

$\hat{\lambda} = \frac{\phi(Z\hat{\beta})}{\Phi(Z\hat{\beta})}$  (or inverse Mills ratio) is then included as regressor in the wage equation. In the reduced form participation equation, both the number of children and net non-labor income appear to have a statistically significant negative impact on labor force participation.

**Table 3 . Maximum likelihood estimates of the step-wise model**

<b>Variables</b>	<b>Coefficients</b>	<b>Estimates</b>	<b>t-values</b>
<b>Exponents:</b>			
Consumption	$\alpha_1$	0.9538	247.6
Leisure	$\alpha_2$	0.7829	48.5
<b>Leisure:</b>			
Constant	$b_0$	-2.6801	-31.9
Age/10	$b_1$	-0.0175	-0.5
(Age/10) <sup>2</sup>	$b_2$	0.0056	1.3
Children 0-2	$b_3$	0.0709	7.9
Children 3-6	$b_4$	0.0108	1.1
Children 7-17	$b_5$	0.0191	4.5
Log-likelihood		-16921.2	
<b>Wage equation:</b>			
Constant	$\gamma_0$	-2.5378	-38.6
Years of educ./10	$\gamma_1$	0.4208	14.3
Experience/10	$\gamma_2$	0.0535	1.5
(Experience/10) <sup>2</sup>	$\gamma_3$	-0.0047	-0.6
Selection term	$\lambda$	0.0289	0.4
Standard deviation	$\sigma_1$	0.3350	
$\sqrt{\text{var}(\varepsilon_2 - \varepsilon_1)}$	$\sigma_4$	0.1186	35.6
No of observations		2,404	

## 6. Elasticities and tax simulations

We use our estimate of the model to compute the utility levels for the individuals as a function of hours of work:

$$(8') \quad U(h) = U_1(h) + e^{\varepsilon_2} U_2(h) = U_1(h) + e^{0.3975 e_i} U_2(h)$$

where

$$(9') \quad U_1(h) = \frac{(C(h) - C_0)^{0.9535} - 1}{0.9535}$$

$$(10') \quad U_2(h) = e^{X_2 \hat{b}} M \frac{\left(1 - \frac{h}{M}\right)^{0.5982} - 1}{0.5982}$$

where

$e_i$  is standard normal(0,1)

Because the wage contains a random part and enters into disposable income we have also to employ the wage equation for all persons, working and not working.

$$(19') \quad \log W = X_1 \hat{\gamma} + \varepsilon_i = X_1 \hat{\gamma} + \hat{\sigma}_1 e_i = X_1 \hat{\gamma} + 0.3365 e_i$$

For each individual we make a draw of  $e_i$  and then the Equations (8')-(10'), together with (19'), is used to find the optimal hours that yield the highest utility. The optimal hours could be negative or zero, which means that it is optimal not to work, otherwise the simulation yields the optimal positive hours of work.

We have calculated the impact of a 1% increase in the wage level on the probability of working (the extensive margin) and on hours worked, given working (the intensive margin). The results are that the average elasticities at the extensive margin is 1.2 and on the intensive margin is 0.65.

In the tax simulations we assess the impact of replacing the tax system of 2001 by the less progressive tax system of 2006 on optimal hours, income and household welfare. To perform the simulations we do not need to smooth the step-wise tax function; we can simply use it as it is. Table 4 reports the average values.

Labor market participation goes down as a result of the change in the tax schedule. But, conditional on participation, the expected number of supplied hours increases. As a result, the unconditional expected number of annual hours

of work, including the zero hours for women who do not work, decreases slightly, from 1,540 to 1,528.

In Table 5 we show the predicted values of disposable income and we observe that households belonging to the upper deciles benefit from having the tax rules of 2001 replaced by the less progressive tax rules of 2006. The Gini coefficient related to the distribution of predicted income based on 2001 tax rules is 0.2387 while the Gini coefficient related to tax rules of 2006 is 0.2694. The tax reform of 2006 thus increases the income inequality.

**Table 4. Labor market participation and annual hours supplied, given working, under the tax rules of 2001 and 2006. Average values.**

	Tax function 2001	Tax function 2006
Participation	0.8727	0.8358
Hours supplied given working	1,765	1,828
Hours supplied in the total population	1,540	1,527

**Table 5. Predicted disposable income by decile – tax system 2001 and 2006. Labor income is due to the modelled behaviour while net capital income is the observed values (inflated from 2001 to 2006). NOK/1000.**

Decile	Freq.	Tax system 2001		Tax system 2006	
		Mean	Std. Dev.	Mean	Std. Dev.
1	241	6.43	5.72	4.64	4.26
2	240	110.60	55.65	66.52	61.32
3	241	175.07	4.87	174.74	5.38
4	240	190.20	4.53	191.01	4.89
5	240	203.41	3.01	204.55	2.86
6	241	213.98	3.41	215.36	3.55
7	240	230.81	5.95	233.33	6.12
8	241	249.17	4.47	256.15	7.94
9	240	269.92	8.49	282.91	8.68
10	240	347.09	49.51	362.48	51.95
Total	2,404	199.60	90.61	199.11	100.62



## **7. Conclusion**

We have shown how a neoclassical labor supply model with optimal decisions (participation and hours of work) derived from first order conditions can be taken to data even in the presence of a step-wise linear progressive tax system which may imply non-convex budget sets. The main contribution of our approach with respect to the pre-existing literature is the fact that we do not need to resort to the traditional but rather restrictive assumption of linear supply curves in empirical neoclassical labor supply. Moreover, our approach is also able to handle non-convex budget sets. Our approach is an alternative to the discrete choice approach to empirical labor supply, which also allows for non-linear labor supply curves and non-convex budget sets. Unlike in the discrete choice approach we do not need to discretize hours when estimating the model.

The labor supply model for married women is estimated on a unique restricted-access dataset. We use the 2001 wave and estimate our model through a one-step full maximum likelihood procedure, which is computationally faster than the traditional stepwise Heckman procedure. Once the model has been estimated we randomly draw the correlated random parts in preferences and wages for each individual and use the model with these draws to calculate optimal hours before and after wage and tax rule changes.

The tax simulations show that to replace the tax system of 2001 with the less progressive tax schedule of 2006 imply a lower labor market participation among the married women, a higher working load, given participation, and a more uneven distribution of household income.

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## Appendix A: Tax functions

**Table A1. The tax function in Norway 2001, Current NOK**

Income NOK (Y)		Tax function
0 -	22,600	0.00
22,600 -	32,267	$0.25 \times Y - 5,550$
32,267 -	60,600	$0.078 \times Y$
60,600 -	144,545	$0.358 \times Y - 16,968$
144,545 -	183,182	$0.2964 \times Y - 8,064$
183,182 -	289,000	$0.358 \times Y - 19,348$
289,000 -	793,200	$0.493 \times Y - 58,363$
793,200 -	above	$0.553 \times Y - 105,955$

As of April 2015, 8NOK are equivalent to 1USD.

**Table A1. Subsistence levels for Consumption**

Year	Minimum consumption ( $C_0$ )
2001	NOK 51,360
2006	NOK 54,955

*Note:* The minimum subsistence level for consumption in 2006 has been set equal to the corresponding 2001 level increased by 7%.

**Table A2. The tax function in Norway 2006, Current NOK**

Nominal income (Y)		Tax (T)
0 -	29,600	0.00
29,600 -	43,023	$0.25 \times Y - 7,400$
43,023 -	67,200	$0.078 \times Y$
67,200 -	93,529	$0.358 \times Y - 18,816$
93,529 -	179,706	$0.2628 \times Y - 9,912$
179,706 -	394,000	$0.358 \times Y - 27,020$
394,000 -	750,000	$0.448 \times Y - 62,480$
750,000 -	above	$0.478 \times Y - 84,980$

In the simulations the nominal values of 2001 have been increased by 7% to make them comparable to 2006 values.

### Tax function 2001 interpolation

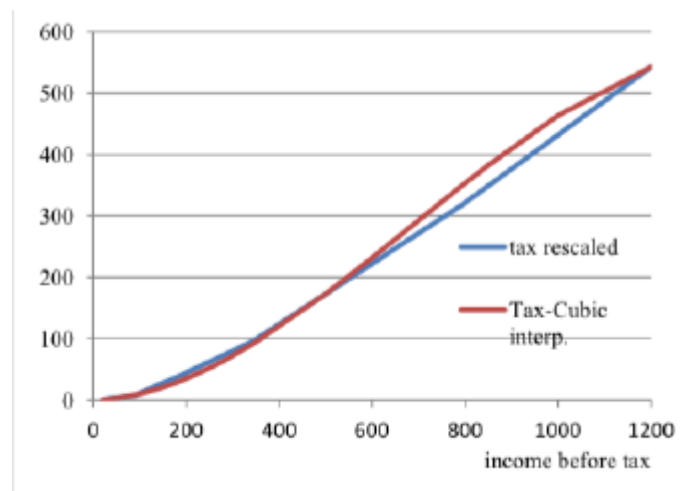
We resort to the cubic function  $T = \alpha_1 G + \alpha_2 G^2 + \alpha_3 G^3$

where T = tax, G = income before tax at the NOK values 70, 500, 1200 (income scaled by 1000).

Applying the tax function 2001 reported in Appendix A, we calculate  $\alpha_1, \alpha_2$  and  $\alpha_3$  that are respectively 0.019818775, 0.000860338, -4.16823E-07.

The interpolation function fit quite well the tax function in the range spanned by the data as shown in Figure A.1.

Figure A.1. Tax according to tax rule 2001 (income rescaled x1000) and tax cubic interpolation



### **Appendix B. Step-wise estimation of the labor supply model.**

To compare the joint estimation of the labor supply model with a step wise estimation we have estimated the latter model. First we estimate a reduced form participation probability,  $\Phi(Z\beta)$  where  $Z=\{1, \text{age}/10, (\text{age}/10)^2, \text{number of children in age } 0-3, 4-6, 7-17, \text{experience}/10, (\text{experience}/10)^2, \text{non-labor income}\}$ . Then the selection term  $\lambda = \frac{\phi(Z\hat{\beta})}{\Phi(Z\hat{\beta})}$  is included as a regressor when the wage equation is estimated. Here  $\phi(\cdot)$  is the standard normal density function and  $\Phi(\cdot)$  is the c.d.f.

Then the wage equation, extended with a selection term,  $\lambda_i$ ,

$$\text{Log}W_i = X_i\gamma + \alpha\lambda_i + \varepsilon_i$$

is estimated.

Let the estimates be  $\hat{\gamma}$  and  $\hat{\sigma}_1$ , where the latter is the estimate of the standard deviation in the wage equation.

Let

$$\hat{W}_{ik} = \exp(X_{li}\hat{\gamma} + \hat{\sigma}_1\eta_{ik})$$

$k$  is random draw  $k$  from a standard normal.  $K$  is the total number of draws, say  $K=20$

where  $\{\eta_{ik}\}$  are iid normal draws from the standard normal distribution.

In this case the likelihood function for those who work has the form

$L_2 =$

$$\prod_{i \in S_2} \left\{ \frac{1}{K} \sum_k \left[ \frac{1}{\sigma_4} \Phi' \left( \frac{\log \partial_1 g(h_i \widehat{W}_{ik}, I_i) + (\alpha_1 - 1) \log g(h_i \widehat{W}_{ik}, I_i) + X_{1i} \hat{\gamma} - X_{2i} b + (1 - \alpha_2) \log \left( 1 - \frac{h_i}{M} \right)}{\sigma_4} \right) \right] \right. \\ \left. \cdot \left| \frac{\alpha_2 - 1}{M - h_i} + \frac{\widehat{W}_{ik} \partial_1^2 g(h_i \widehat{W}_{ik}, I_i)}{\partial_1 g(h_i \widehat{W}_{ik}, I_i)} + \frac{(\alpha_1 - 1) \widehat{W}_{ik} \partial_1 g(h_i \widehat{W}_{ik}, I_i)}{g(h_i \widehat{W}_{ik}, I_i)} \right| \right\}$$

where  $\sigma_4^2 = \text{Var}(\varepsilon_{2i} - \varepsilon_{1i})$ .

The likelihood for non-working individuals equals  $\partial_1 g(0, I_i) = 1$

$$L_1 = \prod_{i \in S_1} \Phi \left( \frac{\log \partial_1 g(0, I_i) + (\alpha_1 - 1) \log g(0, I_i) + X_{1i} \hat{\gamma} - X_{2i} b}{\sigma_4} \right).$$

The parameters to be estimated are  $\alpha_1, \alpha_2, b_1, \sigma_4$ .