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INNOVATION AS AN EMERGING SYSTEM PROPERTY: AN AGENT BASED MODEL

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ABSTRACT

The paper elaborates the notion of innovation as an emerging property of complex system dynamics and presents an agent-based model of an economy where systemic knowledge interactions among heterogeneous agents are crucial for the generation of new technological knowledge and the introduction of innovations. In this approach external knowledge is an indispensable input, together with internal learning and research activities, into the generation of new knowledge. The introduction of innovations is analyzed as the result of systemic interactions among myopic agents that are credited with an extended procedural rationality that includes forms of creative reaction. The creative reaction of agents may lead to the introduction of productivity enhancing innovations. This takes place only when the structural and institutional characteristics of the system are such that agents, reacting to out-of-equilibrium conditions, can actually take advantage of external knowledge available within the innovation system into which they are embedded. Building upon agent-based simulation techniques the paper explores the effects that alternative configurations of the intellectual property right regimes play in assessing the chances to generate new technological knowledge and shows how the different architectural configurations of the structure into which knowledge interactions take place affect the rates of introduction of technological innovations. The results of the simulation model suggest that the dissemination of knowledge favors the emergence of creative reactions and hence faster rates of introduction of technological innovations.

KEY-WORDS: Complex System Dynamics; Innovation; Emergent Property; Technological Knowledge; Intellectual Property Rights; Knowledge Dissemination; Agent-based Simulation.

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1. Introduction

The paper presents an agent-based model of an economic system where innovation is characterized as the emergent property of the system dynamics of knowledge interactions. The introduction of innovations is analyzed as the possible result of systemic interactions among heterogeneous and myopic, yet learning, agents that can take advantage of external knowledge so as to make their reaction, creative, as opposed to adaptive.

Agents are myopic: their rationality is bounded, as opposed to Olympian, because of the wide array of unexpected events, surprises and mistakes that characterize their decision making and the conduct of their business in a ever changing environment. Our agents, however, are endowed with an extended procedural rationality that includes the capability to learn and to try and react to the changing conditions of their economic environment by means of the generation of technological knowledge and its exploitation by means of the introduction of technological innovations. In this approach agents do more than adjusting prices to quantities and vice versa: they can try and change their technologies. Agents are intrinsically heterogeneous. Their basic characteristics differ in terms of original endowments such as learning capabilities, size, and location. Their variety is also endogenous as it keeps changing as a result of the dynamics of endogenous technological change.

The actual chances that they can succeed in such a creative endeavor and actually introduce technological innovations however depend upon the institutional and structural characteristics of the system into which they are embedded. The reaction of agents may lead to the introduction of productivity enhancing innovations when they can actually take advantage of external knowledge available within the innovation system into which they are embedded. In this approach external knowledge is an indispensable input, together with internal research activities, into the generation of new knowledge.

The aim of the paper is to show how the different institutional and architectural settings of the structure into which knowledge interactions take place affect the rates of generation of new knowledge and the pace of introduction of technological innovations.

The rest of the paper is structured as it follows. Section 2 elaborates the theoretical framework and presents the building blocks of an approach that integrates the economics of innovation and the economics of complexity. Section 3 presents the agent-based frame of the innovation system. Section 4 exhibits the results of the simulation focusing upon the alternative hypothesis about the institutional and architectural features of the innovation system. The conclusions summarize the main results and put them in perspective.

2. The theoretical frame

2.1. The general hypotheses

In our approach, innovation stems from the reaction of firms to the changing conditions of both factor and product markets. Following the late Schumpeter (1947) firms' reaction to the changing condition of their economic environment that engenders out-of-equilibrium conditions can be either adaptive or creative². Reaction can be just adaptive and lead to traditional price/quantity adjustments with no innovation or actually creative. Appropriate structural and institutional characteristics of the system may engender the introduction of productivity enhancing innovations. Innovations emerge when a number of complementary conditions at the system level apply. This approach enables to solve the conundrum of the economics of innovation: if innovation is the result of the intentional decision making of each individual firm in isolation there is no way to explain why total factor productivity should increase. Firms should in fact increase the amount of innovative efforts to the point where their marginal output matches their marginal costs. If this takes place, of course, no total factor productivity increase might take place.

Only when the role of external and complementary systemic conditions is taken into account the role of innovation as the productivity enhancing result of an intentional action can be articulated. When the role of the external context is properly appreciated it becomes clear that innovation is not only the result of the intentional action of each individual agent, but it is also the endogenous product of dynamics of the system. The individual action and the system conditions are crucial and complementary ingredients to explain the emergence of innovations.

² Schumpeter (1947) makes the point very clear: "What has not been adequately appreciated among theorists is the distinction between different kinds of reaction to changes in 'condition'. Whenever an economy or a sector of an economy adapts itself to a change in its data in the way that traditional theory describes, whenever, that is, an economy reacts to an increase in population by simply adding the new brains and hands to the working force in the existing employment, or an industry reacts to a protective duty by the expansion within its existing practice, we may speak of the development as an *adaptive response*. And whenever the economy or an industry or some firms in an industry do something else, something that is outside of the range of existing practice, we may speak of *creative response*. Creative response has at least three essential characteristics. First, from the standpoint of the observer who is in full possession of all relevant facts, it can always be understood *ex post*; but it can be practically never be understood *ex ante*; that is to say, it cannot be predicted by applying the ordinary rules of inference from the pre-existing facts. This is why the 'how' in what has been called the 'mechanisms' must be investigated in each case. Secondly, creative response shapes the whole course of subsequent events and their 'long-run' outcome. It is not true that both types of responses dominate only what the economist loves to call 'transitions', leaving the ultimate outcome to be determined by the initial data. Creative response changes social and economic situations for good, or, to put it differently, it creates situations from which there is no bridge to those situations that might have emerged in the absence. This is why creative response is an essential element in the historical process; no deterministic credo avails against this. Thirdly, creative response –the frequency of its occurrence in a group, its intensity and success or failure- has obviously something, be that much or little, to do (a) with quality of the personnel available in a society, (b) with relative quality of personnel, that is, with quality available to a particular field of activity relative to the quality available, at the same time, to others, and (c) with individual decisions, actions, and patterns of behavior." (Schumpeter, 1947:149-150).

Innovation cannot be considered but the intentional result of the collective economic action of agents. Innovation is the endogenous result of the system dynamics and it does not fall from heaven, as standard economics suggests. Neither is it the result of random variation as evolutionary approaches contend. Dedicated resources to knowledge governance are necessary to implement the competence accumulated by means of learning and to manage its exploitation. Agents succeed in their creative reactions when a number of contingent external conditions apply at the system level. Innovation is made possible by key systemic conditions: “innovation is a path dependent, collective process that takes place in a localized context, if, when and where a sufficient number of creative reactions are made in a coherent, complementary and consistent way. As such innovation is one of the key emergent properties of an economic system viewed as a dynamic complex system” (Antonelli, 2008:I).

The appreciation of the systemic conditions that shape and make innovations possible, together with their individual causes leads to the identification of innovation as an emergent property of a system. This approach provides a solution to the conundrum of an intentional economic action whose rewards are larger than its costs, only if the organized complexity that enables the emergence of innovations is explained as an endogenous and dynamic process engendered by the interactions of rent-seeking agents, that try and cope with the ever changing conditions of their product and factor markets (Antonelli, 2009 and 2010).

Following a well-established line of analysis agents are characterized by bounded rationality and learning capabilities. These agents are rooted in a well-defined set of characteristics that stem from the quasi-irreversibility of their tangible and intangible inputs, including their location in the multidimensional space. At each point in time, however, if and when out-of-equilibrium conditions take place, agents can react and switch, i.e. change the structure of their inputs and their location, but only with the investment of dedicated resources. Specifically agents, at each point in time, can change, within a limited ray, their knowledge, their technology and the structure of their interactions. Technological change is inherently localized: each agent can innovate, but only in the surroundings of its original multidimensional location, in technical space, when positive feedbacks in regional and knowledge space are at work. Hence agents are heterogeneous. They are characterized by distinctive and specific characteristics that qualify their competence, the endowment of tangible and intangible inputs and their location in the space of interactions (Cyert and March, 1963; March, 1988).

The introduction of technological and organizational innovations requires the generation of new knowledge. The generation of knowledge is characterized by specific attributes: knowledge is at the same time the output of a specific activity and an essential input into the generation of new knowledge. Because of knowledge indivisibility and specifically, because of diachronic knowledge cumulability and

synchronic knowledge complementarity, the access to existing knowledge, at each point in time, is a condition necessary for the generation of new knowledge. Yet no firm can command all the available knowledge, hence no firm can generate new technological knowledge alone. The twin character of knowledge as an output of a research process and the input into the generation of further knowledge stresses the basic complementarity and interdependence of agents in the innovation process: innovations is inherently the collective result of the interdependent and interactive intentional action of economic agents (Blume, Durlauf, 2001 and 2005).

The structure of the system and its continual change, following the tradition of analysis of Simon Kuznetz, play a crucial role. The architecture of knowledge externalities, interactions and transactions plays in fact a crucial role in the access to external knowledge and hence in the definition of the actual chances of agents to implement their reactions and make it creative, as opposed to adaptive.

Technological knowledge is viewed as the product of recombination of existing ideas, both diachronically and synchronically. The generation of new knowledge stems from the search and identification of elements of knowledge that had not been previously considered and their subsequent active inclusion and integration with the preexisting components of the knowledge base of each firm (Weitzman, 1996 and 1998; Fleming and Sorenson, 2001).

Positive feedbacks take place when the external conditions into which each firm is localized, qualify the reaction of each firm, and make the access to external knowledge possible so as to make the reaction of firms creative, as opposed to adaptive. When the access conditions to the local pools of knowledge make possible the actual generation of new technological knowledge and feed the introduction of innovations, actual gales of technological change may emerge. The wider is the access to the local pools of knowledge and the larger is the likelihood that firms are induced to react. The larger the number of firms that react and the better the access conditions to external knowledge and the stronger are the chances that their reaction are creative: technological change becomes a generalized and collective process (Arthur, 1990 and 1994).

Marshallian externalities as implemented by the notion of generative interactions play a central role in this approach (Lane and Maxfield, 1997). The amount of knowledge externalities and interactions available to each firm influences their capability to generate new technological knowledge, hence the actual possibility to make their reaction adaptive as opposed to creative and able to introduce localized technological changes. Each myopic agent has access only to local knowledge interactions and externalities, i.e. no agent knows what every other agent in the system at large knows. Because of the localized character of knowledge externalities and interaction, location in a multidimensional space, in terms of distance among agents and their density, matters. Agents are localized within networks of transactions and interactions that are

specific subsets of the broader array of knowledge externalities, interactions and transactions that take place in the system.

In such a context innovation is an emergent property that takes place when complexity is 'organized', i.e. when a number of complementary conditions enable the creative reaction of agents and makes it possible to introduce innovations that actually increase their efficiency. The dynamics of complex systems is based upon the combination of the reactivity of agents, caught in out-of-equilibrium conditions, with the features of the system into which each agent is embedded in terms of externalities, interactions, positive feedbacks that enable the generation of localized technological change and lead to endogenous structural change (Anderson, Arrow, Pines, 1988; Arthur, Durlauf, Lane, 1997).

2.2 The two knowledge trade-offs

In this context, because of the twin character of knowledge as the output of a research process and the input into the generation of further knowledge, two knowledge dissemination trade-offs take place. The first relates to the structure of intellectual property right regimes; the second to the distribution in economic, regional and knowledge space of knowledge generation activities. Let us analyze them in turn:

A) The intellectual property right trade-off. The structure of the intellectual property right regimes, the scope of patents, their duration, the assignment procedures and their exclusivity play a crucial role. Strong intellectual property right regimes increase the appropriability of technological knowledge for they limit the leakage of information and delay uncontrolled imitation. Innovators can secure for a longer period of time the benefits stemming from the generation of new technological knowledge and the introduction of technological knowledge. Strong intellectual property regimes increase the chances of innovators to exploit technological knowledge. Consequently strong intellectual property right regimes enhance the incentives to the generation of new knowledge and hence help increasing the amount of resources that would be committed to the generation of new knowledge. Strong intellectual property right regimes, however, reduce both the static and the dynamic efficiency of economic and innovation systems. Strong property right regimes increase the duration of monopolistic power in the product markets and the appropriation of consumers' surplus by innovative suppliers. Strong property right regimes, moreover, reduce the dynamic efficiency of innovation systems because they prevent and delay the access to existing knowledge as an input into the generation of new knowledge. The combined effect of strong property right regimes in fact is to increase the incentives to generate research and hence the amount of resources but the reduction of their efficiency because at each point in time available knowledge cannot be used to generate new knowledge and must be invented again. Strong intellectual property right regimes risk to increase the replication of research efforts and the reduction of the pace of generation of technological knowledge. This knowledge trade-off requires the fine-tuning of intellectual property rights with the

identification of the proper mix of the protection of appropriability on the one hand and the dissemination of available knowledge.

B) The architectural trade-off. The architectural characteristics of the network of interactions that qualify each economic system have powerful consequences on the actual capability of each economic agent to generate new technological knowledge. The distribution in regional and knowledge space of knowledge generation activities has important effects. Because of the pervasive role of external knowledge as an input into the generation of new technological knowledge the regional concentration of knowledge generating activities may increase the pace of technological advance. Proximity, in fact, helps the identification of useful external knowledge hence reduces search and exploration costs. Proximity in regional space helps reducing the risks of opportunistic behaviors because of increased interactions, hence helps limiting transaction costs and finally proximity increases the homogeneity of codes and favors the absorption of external knowledge. Excess concentration may favor the forging ahead of small but effective clusters of highly innovative groups of firms strongly interconnected and able to interact at a fast pace. At the same time, however, excess concentration might be identified where the rest of the system is cut of the flows of creative interactions and the dissemination of new knowledge is delayed. Excess concentration risks to reduce knowledge variety and the related opportunities for knowledge recombination. The dissemination of knowledge generating activities may help the stimulus to the generation of new knowledge because of the wider participation of a larger variety of agents in the collective endeavor that leads to the generation of new knowledge. Once more it is clear that a knowledge trade-off between concentration and dissemination of knowledge generating activities takes place with important policy implications about the best allocation of additional research resources and activities through regional space.

Agent based models can help structuring in a rigorous frame of analysis the dynamic properties of the system so as to provide a context into which the implementation of simulation techniques can exhibit the different results of alternative structures of knowledge interaction mechanisms and intellectual property rights regimes³. This exercise can contribute the implementation of an approach that adapts complex system dynamics to economics where technological change is the central engine of the evolving dynamics of the system and it is the result of the creative response of intentional agents, embedded in an evolving architecture of market, social and knowledge interactions (Aghion, David, Foray, 2009; Terna, 2009).

The simulation of the working of an economic system where technological change can take place implements the basic intuitions of complexity theory and of economics of innovation. The simulation will enable to identify the proper solutions to the two knowledge trade-off that have been identified with respect to the structure of

³ Empirical investigations and tests of specific hypotheses can complement and support agent-based simulations. See Antonelli and Scellato, 2008 and 2009; Antonelli Patrucco Quatraro, 2008.

intellectual property right regimes and the regional distribution of knowledge generation activities.

Let us now turn our attention to analyze the building blocks of our agent-based simulation model. The following section shows how the use of the basic tools of agent-based simulation can implement a rigorous representation of the dynamics of a full-fledged economic system where agents are credited with the capability of generating technological knowledge and generating technological innovations provided a conducive architecture of network interactions and an effective intellectual property right regime is implemented.

3. The simulated economy

3.1. The structure of the model

To investigate the hidden dynamics for the innovation process at a systemic level a very simplified economy has been reproduced into the simulation model. In the system enterprises produce homogeneous products sold into a single market; production inputs too are bought into a single factor market. Into those markets operates an indistinct bunch of agents that represent both factor suppliers, mainly workers, researcher involved into the innovation field, and shareholders; these agents constitute also the customers population of the firms.

Heterogeneous firms are embedded into an economic structure represented as a physical space that determines their possibility to interact among others, to observe their results and, eventually, copy their technologies. Depending on their technological level they are inserted in a technological space too. Both spaces are managed as grids divided into cells each one can hosts an unlimited number of enterprises.

For the sake of simplicity, no financial institutions have been activated, neither payments can be postponed. The whole capital of the enterprise is supplied by the shareholders and all the commercial transactions are immediately regulated.

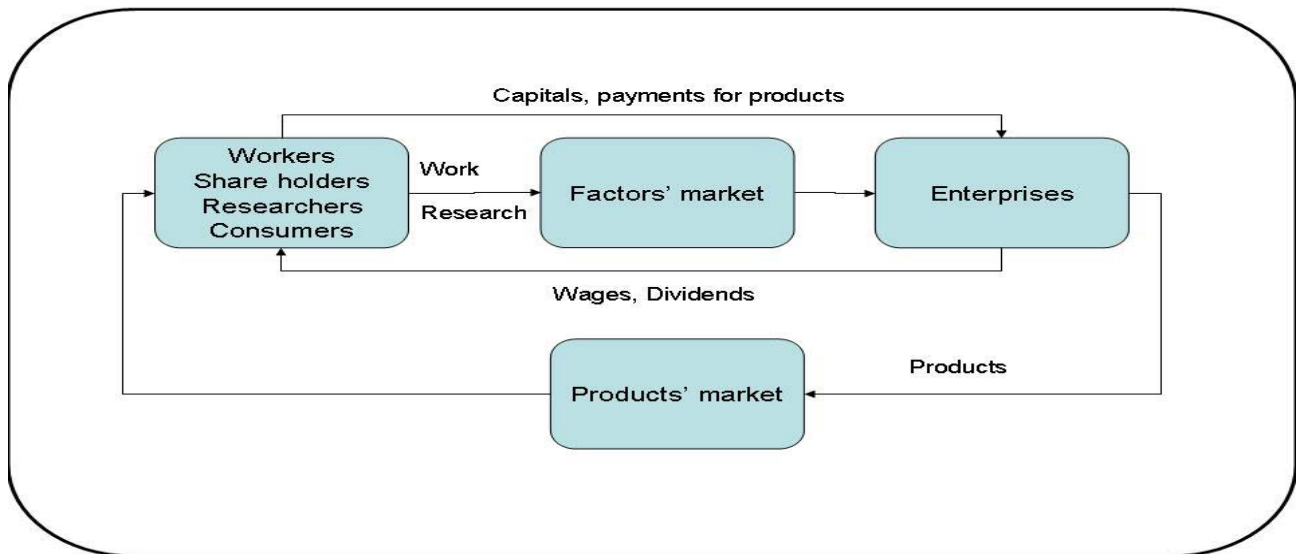


Figure 3.1 Fluxes into the market

Market clearing mechanisms based exclusively upon prices maintain a perfect equilibrium between demand and supply. Such an equilibrium is ensured for both the product and the factor market too: the quantities determine the correct price to sell all the production. No friction neither waiting times are simulated, factors are immediately available.

The production function is very simple, in order to avoid matters related to different kinds of production processes, inputs availability, warehouses cycles and so on: outputs depend exclusively from the amount of employed work and its productivity, following a simple linear function like:

$$3.1 \quad O_i = \pi_i L_i.$$

Where the output (O), of a generic i-th enterprise, depends from the employed labour (L) and its productivity (π), that could vary between zero and one. Both labour and productivity may vary among the enterprises: labour depends on the entrepreneur's decision about the growth of the production. Productivity is a function of the technological level the firm achieved through innovation.

The whole product of all enterprises is totally sold on the single market, the price depends on the liquidity of the market. Customers (i.e. workers, share holders and researchers) spend the whole amount they earn in buying goods, so the selling price for goods is simply computed as:

$$3.2 \quad p = \frac{Y}{\sum_{i=1}^n O_i}.$$

Where Y represents the whole amount earned by the customers and n is the number of enterprises operating into the simulated economy. If W denotes the amount of wages, R the expenses for research and D the dividends:

$$3.3 \quad Y = W + R + D.$$

The cost for single work unit is the same for each enterprise; it is centrally computed as a linear function:

$$3.4 \quad w = 50 + 0.005 \sum_{i=1}^n L_i.$$

Each enterprise pays its workers a total amount of wages of:

$$3.5 \quad W_i = wL_i$$

The whole amount of wages is simply computable as:

$$3.6 \quad W = \sum_{i=1}^n W_i$$

The research costs are directly related to the actions performed by each enterprise to innovate, either:

- consolidate own experiences,
- explore novel solutions to solve existing problems,
- imitate neighbours,
- move to other location in order to exploit more developed neighbours.

By performing such actions the enterprises change their position into the technological space or the physical one. The research costs is directly related to the distance between the old and the new position of the enterprise in each space:

$$3.7 \quad R_i = dT_i + dF_i.$$

Where dT and dF are the technological and physical distance the enterprise covered in its innovation process. The whole amount research suppliers receive is:

$$3.8 \quad R = \sum_{i=1}^n R_i.$$

All the enterprises redistribute the whole profit to the shareholders, but they are obliged to immediately reintegrate each loss. Profits are computed as difference between income and costs, no taxes are to be paid, neither part of the profit can be retained into the enterprise. Naming P the profit of a generic enterprise gives the following equation:

$$3.9 \quad D_i = P_i = pO_i - W_i - R_i.$$

Where D could be less than zero if a loss had to be reintegrated. The amount of dividends paid to the whole systems is:

$$3.10 \quad D = \sum_{i=1}^n D_i.$$

At the aggregate level the system could be resumed by substituting into the 3.3 the expressions 3.6, 3.8 and 3.10 to obtain:

$$3.11 \quad Y = \sum_{i=1}^n W_i + \sum_{i=1}^n R_i + \sum_{i=1}^n D_i.$$

By specifying D_i using the expression 3.9 it is possible to obtain:

$$3.12 \quad Y = \sum_{i=1}^n W_i + \sum_{i=1}^n R_i + \sum_{i=1}^n pO_i - \sum_{i=1}^n W_i - \sum_{i=1}^n R_i.$$

By operating simple compensations the 3.12 becomes:

$$3.13 \quad Y = \sum_{i=1}^n pO_i.$$

Recalling the expression 3.2 it is evident that the whole system could reach equilibrium and the amount of money into the system remains always constant.

As briefly introduced before, enterprises are located into a physical space, simulated by a grid, each enterprise lays into a cell but cells are able to pile more than one enterprise. The position into the grid determines the neighbourhood into which firms can observe their competitors, comparing results and copying public technological solutions, the model indeed is able to manage patent rights in order to increase the plausibility of the simulation.

The technological level of each enterprise determines its position into the technological space, another grid where enterprises are inserted accordingly with their innovation level. The position into the technological space determines the firm's productivity: the grid is 100 cells wide, both horizontally and vertically, the productivity (π) grows toward the upper right corner of the space following a trivial rule:

$$3.14 \quad \pi_i = \frac{tX_i + tY_i}{200}.$$

Into the 3.14 formula tX and tY represent the, horizontal and vertical, technological position of the i -th enterprise; innovation means increasing the productivity by moving up the cumulate X and Y position.

Note that enterprises technologically very close could be positioned in far distant cells into the physical space and vice versa. In this way imitation of physically close firms may enable the introduction of an innovation with positive effects in terms of productivity growth.

3.2 Model dynamics

The model is based upon a precise sequence of actions to be performed by the different operators: results obtained during a production and consumption cycle influence the strategies the agents will take during the next cycle. The dynamics of the model is typically characterized by path dependence: the dynamics in fact is non-ergodic because history matters and irreversibility plays a role. At each point in time, however, occasional events may alter the 'path' i.e. the direction and the pace of the dynamics (David, 2007).

The model reproduces each production cycle by starting with the provisioning of factors to end computing profits and paying dividend, wages and research services. First each enterprise sends to the market its request of work units; in this model workers are extremely flexible, they could work for different enterprises few hours for each. The amount of work force each firm will employ depends on the results it has obtained during the previous cycle: if the firm has just obtained a profit it will try to expand its production by rising the number of work unit, as well as in case of loss it will reduce the usage of work. After all the enterprises have sent their order to the market, it computes the price for work unit that will be the same for all the enterprises:

$$3.15 \quad L_i^t = f(P_i^{t-1}).$$

$$3.16 \quad w^t = 50 + 0.005 \sum_{i=1}^n L_i^t$$

The production of each firm depends on the amount of work it employs and its own productivity, so:

$$3.17 \quad O_i^t = \pi_i^t L_i^t.$$

After producing the enterprises send to the market the amount of product they intend to sell, as well as the consumers send to the market the amount of money they intend to spend; using such data the market is able to compute the price for a single product unit, that will be cashed by all the enterprises. Note that, because the decision about how much research to do is taken by the enterprises only at the end of the production cycle, the total amount of money consumers are given in payment for research depends on the decision taken two times before.

$$3.18 \quad p^t = \frac{W^{t-1} + D^{t-1} + R^{t-2}}{\sum_{i=1}^n O_i^t}.$$

The production cycle ends with the enterprises cashing the incomes for sold products, paying wages and research services, computing the profits, and distributing dividends or collecting money from the shareholders to face losses.

Enterprises compare their results with the average profit obtained by their neighbours: if their results are lower than the average they do some research to improve their productivity. Assuming that each firm has a certain number of neighbours (m), this process could be resumed as:

$$3.19 \quad R_i^t = f\left(P_i^t, \frac{\sum_{i=1}^{i-1} P_i^t + \sum_{i=1}^{i+m/2} P_i^t}{m}\right).$$

3.3 The strategies to innovate

The enterprises of the model are endowed with the ability to learn better ways to perform their production cycles. Each time a production cycle is done, firms acquire and cumulate some technological potential (tp). Such a potential could be transformed in real innovation by affording the appropriate research costs with the appropriate access to external knowledge.

The notion of recombinant knowledge plays here a central role. As Weitzman (1996: 209) recalls: “new ideas arise out of existing ideas in some kind of cumulative interactive process that intuitively has a different feel from prospecting for petroleum”. Recombination may be both vertical and horizontal. When the former takes place new ideas are generated by means of the diachronic recombination. When the latter applies, instead, new ideas are product of synchronic (Weitzman, 1998).

This insight leads to articulate the view that new ideas are being generated through the recombination of existing ideas, under the constraint that the access to existing ideas is possible.

Firms are given the capability to observe and, eventually, imitate their neighbours. Imitation, however, because of absorbing costs is not free. In order to increase the plausibility of the model enterprises can imitate only the technologies that are similar to their own, in other words they cannot pass from a very low technological level to a high one directly (Patrucco, 2009).

A major limitation to the possibility to take advantage and copy others' technologies is represented by intellectual property rights (IPR). In order to model a credible IPR regime we allow enterprises to patent their technology and hence to retain exclusive exploitation rights for a certain number of cycles (Reichman, 2000).

Because the possibility to observe neighbours depends on the position of each enterprise into the physical space, when imitation gives poor or null results enterprises could decide to move into another location in order to meet better technological conditions.

Here we see how the structure of the system influences in several ways the innovation chances of the enterprises: learning is faster for firms that operate in a well developed neighbour, and imitators have higher possibilities to observe and copy if they operate into a crowded and technologically advanced environments (Ozman, 2009).

Accumulation of experience proceeds at a specific "learning rate" (lr) that represents the fraction of an innovation unit the potential is added for each production cycle. The learning rate is the same for all the enterprise and is managed as a model's parameter, so different values for it could be experimented.

Accordingly with another model parameter, called "learning factor" (lf), the learning rate is biased by the maturity level of the enterprises' neighbourhoods, measured as the average productivity of the neighbours enterprises (π_n).

Firms operating in a neighbourhood whose average productivity is greater than 0.5 (the average level for the model's productivity range as sub 3.1) are able to increase their potential faster than the learning factor, as well as learning of firms included in lower productive neighbourhoods is less than the learning factor.

Equation 3.20 describes the algorithm used to compute learning:

$$3.20 \quad tp_i^t = tp_i^{t-1} + lr * lf * (\pi_i^t - 0.5)^2.$$

The accumulated potential could be transformed in real innovation when almost one unit is achieved: to innovate the enterprises move from the original technology to another one that is one technological unit greater and increase the productivity by 1/200. Such enhancement has a cost proportional to the enhancement that reduce the profit for the incoming exercise.

If the cumulated potential is not enough to support an innovation, firms could observe the technology their neighbours are using and try to imitate. Several model's parameters are employed to regulate this activity.

Because of bounded rationality, firms can observe only the other ones that lay in a certain neighbourhood whose extension depends on the "view" (v) parameter: this value limits the number of positions all around the agent it can explore. Due to the fact the simulated world is managed as a grid the position of the agent could bias this view: agents in a corner have less possibility to observe than other located in the middle of the grid, as well as agents in a very crowded neighbours have more information than isolated firms. Note that a single position into the grid could pile several agents, so simply exploring its cell an agent may found other firms to observe.

The view parameter determines only the number of cells the agent can access, the real number of other firms it can observe depend upon the evolution of the agents' distribution and constitutes an emerging phenomenon that continuously evolves during the execution of the simulation. Saying c to be the number of accessible cells and v the value of the view parameter, each agent can potentially access a number of cells of:

$$3.21 \quad c = (v * 2 + 1)^2.$$

When the agent is located near the end of the grid its capability falls dramatically, for instance the number of cells an agent located in a corner can access is:

$$3.22 \quad c = (v + 1)^2.$$

By observing other firms an enterprise knows the latest technological level they apply that is not covered by a patent licence. A specific model's parameter "patent duration" (pd) is used to experiment different scenarios, its value determines the number of production cycles each innovation remains hidden to the competitors, accordingly with the following formula where pT represents the public technology of a firm and T the private one:

$$3.23 \quad pT_i^t = T_i^{t-pd}.$$

The two values are the same only at the start of the simulation and for firms that did not adopt any innovation during the past production cycles.

Observed technologies can be imitated only if the distance between them and the own ones is less than a parametrical value, so called “imitation threshold”. This limitation has been introduced to avoid dramatic jumps in the productivity of firms that would be not plausible. Imitation has a cost equal to the named distance too.

The third way to innovate consists in moving around the physical space in order to reach more interesting neighbours. When consolidation of the potential and imitation cannot be performed the enterprises move randomly to another location in the hope to found better developed zones. Movement is limited by a parameter called “jump”, its value determines the maximum amount of cells the firms can go through vertically and horizontally back or forward; the effective number of cells the enterprise will move is determined randomly into this range, that constitutes a Von Neumann’s neighbour. Moving costs are equal to the innovating ones.

3.4 The motivation to innovate

The enterprises of the model are always comparing their performances in terms of profits, to the neighbours average results, the difference between own figures and neighbours average ones increases the motivation to innovate. If the results are under the average level, innovation could be used to increase them, as well as when the results are above the average level the positive trend would increase the appeal of new investments. Innovation is viewed as the possible result of intentional decision-making that takes place in out-of-equilibrium conditions. The farther is profitability from the average and the deeper the out-of-equilibrium conditions. The farther away is the firm from equilibrium and the stronger the likelihood for innovation to take place. Hence we assume a U-relationship between levels of profitability and innovative activity, as measured by the rates of increase of total factor productivity. (Antonelli and Scellato, 2009).

Summarizing each time an enterprise’s result is found to be far enough from the average that firm increases its motivation to innovate. Such a motivation become stronger and stronger if the enterprise’s relative position remains outside a band for several and consecutive production cycles: after the third consecutive cycle the enterprise performs an innovation trial.

The comparison between own and neighbours results is biased by a “tolerance” (ξ) value, one of the several model’s parameters, that could vary between zero, that means no tolerance, and infinite, that means maximum tolerance. In this way the enterprise compute a difference able to motivate innovation, only if:

$$3.24 \quad p_i^t < (1 - \xi) \frac{\sum_{j=i-m/2}^{i-1} p_j^t + \sum_{j=i+1}^{i+m/2} p_j^t}{m}.$$

or

$$3.25 \quad p_i^t > (1 + \xi) \frac{\sum_{j=i-m/2}^{i-1} p_j^t + \sum_{j=i+1}^{i+m/2} p_j^t}{m}.$$

When innovation becomes mandatory firms immediately try and consolidate their own potential. The firms that have not sufficient potential try and imitate a neighbour and, if imitation is not possible too, they can move randomly to another location into the physical space.

Even if comparing own and neighbours results had not yet supplied enough motivation to innovate, the firms may try to copy neighbours' solutions, such an action would be performed at random accordingly with a probability that depends upon the patent duration.

It is plausible to expect that the longer is the patent period, value of the patent duration parameter (pd), the higher will be the research effort: unless enterprises were given the exclusive possibility to exploit the research results, no private firms would be interested in investing money, because their discovery would be immediately available for competitors. In the model, even with patent duration equal to zero, the new technology is exploited exclusively by the innovating enterprise for almost one cycle.

Each time an enterprise found it self far from the neighbours, but has not yet reached enough motivation to innovate, it tries to imitate with a probability (ip) of:

$$3.26 \quad ip = \frac{pd}{100}.$$

Note that, in this case, imitation only is performed if there are no chances to imitate, that could only be due to the neighbours are too much technologically advanced, no other strategies are performed.

Innovation, either performed through consolidation or imitation, does not guarantee the profit level will reach the neighbourhood average one, so enterprises could be continuously motivated to innovate.

Imitation may be ineffective in leading the enterprise enough close to the neighbours because the public technologies, the old ones, could be far from what its competitors perform, as well as the raise of the technology level would be too poor.

Moving is worth only to explore the space, it does not give immediately productivity advantages; this strategies could also lead to some weird results: for instance an enterprise that is weak in a neighbourhood so advanced that it can not imitate any neighbours, may move to a zone where its own profit is so close to the average one that it will loose the motivation to innovate.

In the model share holders usually afford losses by investing new capital, but such a behaviour can not be maintained for a long time, so enterprises can afford a limited amount of cumulated loss before closing. After each production cycle enterprises cumulate the profit, or loss, in a counter, when the cumulated amount is greater than a threshold, managed as model's parameter, named "max loss" the enterprise stops its activity and disappears.

Each time an enterprise closes it lives room for a new one that can fill the supply gap; usually this process takes time to be completed, so in the model a dead enterprise is replaced by a new one after a defined number of production cycles, managed by the parameter "revamp time". New enterprises are physically located in the place leaved by the dead ones, but they adopt a technological level equal to the current public average level of the neighbourhood.

3.5 The distribution of knowledge

The enterprises are endowed, at the start of the simulation, with a competence and a technological level, that is randomly tossed for each between zero and the 25% of the maximum achievable level, following a uniform probability distribution. The simulations were started with low skilled firms, with a uniform distribution among them, both to:

- give each firm the possibility to express an own development path,
- give each researcher a similar starting situation to analyze the different development paths.

In the real world competence centres, like universities, technical and management schools and so on, are located on the geographical territory; enterprises operating in geographical regions whit an high density of such organisations could access higher level of knowledge and reach higher technologies.

To introduce these aspects in the simulation the model can manage geographical regions represented by physical spaces where competence can be distributed

following different configuration: from a full concentration in a limited space to a well sparse distribution.

Competence centres are represented by enterprises with very high technological level (so called “genius”), whose initial knowledge endowment is randomly tossed within the highest quarter of the possible values, whereas normal agents are given values in the lowest one.

Genius could be imitated by their neighbours accordingly with the imitation threshold value set up for the simulation, so the higher the imitation possibility is the stronger is the influence of the genius to their neighbours. The patent duration does not slow the effect because the initial knowledge is pretended to be an old and public one.

In order to experiment different scenarios the number of genius is parametrically managed and could be set to zero to exclude this effects.

The distribution in space of agents is tossed randomly at the beginning of the process but it becomes fully endogenous as agents are credited with the capability to move in regional space searching for the access to external knowledge spilling in the proximity of ‘genius’. Hence the dynamics of the regional distribution of agents exhibits the typical traits of path dependence. The process is non-ergodic but small variations can exert important effects in terms of emergence of strong clusters or, on the opposite, progressive dissemination in space (D’Ignazio and Giovannetti, 2006; Antonelli, 2008).

3.6 The working of the system

The model can be considered a very simple and yet representation of an economic system where all relations are fully consistent. All transactions are cleared in the market place. Production and distribution are tied in a coherent way. So far the model presents all the basic features of a simplified general equilibrium system, as opposed to a partial equilibrium one. The walrasian foundations of the system analysis however are radically changed, actually implemented, by the intrinsic heterogeneity of agents, by the endogeneity of technological change and by the path dependent dynamics. Technological change is twice endogenous in this system. First, because the introduction of technological innovations is explicitly made by agents that try and react to their performances. Second, because the outcome of their reaction, whether adaptive or creative, is determined by effects of the architectural and institutional characteristics of the system upon the generation of technological knowledge.

The identification of the key role of technological change as the endogenous result of the interaction between the conduct of myopic agents credited with an extended procedural rationality that includes both learning capabilities and potential creativity, and the characteristics of the system makes it possible to appreciate the path dependent dynamics of the system. Each step in the process, in fact, can be

understood only if the historic sequence of events that has shaped the process is fully understood: the past exerts a strong effect on the future. At the same time, however, the changing structure of the system, produced by the mobility of firms in regional and technological space and by the possible introduction of technological changes that are contingent to the emerging structure, may exert powerful effect upon the system trajectory, both in terms of rate and direction.

Hence the working of the model provides a reliable, comprehensive and coherent account of the evolving dynamics of an economic system where change is not limited to the convergence towards equilibrium from out-of-equilibrium conditions, but is inherent and intrinsic to the system. The system, in fact, does not converge towards a stable and single attractor, as it is the case in the walrasian model, but keeps changing exactly because agents are credited with the capability to generate new technological innovation and introduce technological innovations according to their performances and to the specific conditions of the systems in terms of architecture of the regional distribution and institutional set-up.

It can be considered a platform that can be used to test the systemic implications of different and alternative configurations of the basic parameters. In section 4 we shall explore the effects of such alternative configurations.

Other applications enable to study the dynamics characteristics of the system and provide reliable information about the evolution of firms, industries, and regions in terms of performances such as rates of growth of output, rates of increase of productivity, changes in the concentration and in general of the variety and heterogeneity across firms.

At the aggregate level this model can be used to study the fluctuations of the system and the eventual emergence of cyclical patterns of change and the sensitivity of the cyclical fluctuations to alternative specifications of the model in terms of distribution of the size, profitability and location of firms and the consequent rates of introduction of innovations.

4. Results of the simulations

The strength of simulation technique consists in the possibility to assess in a coherent and structured frame the systemic consequences of alternative configurations of the properties of the system. Simulation techniques allow to exploring the outcomes of different hypotheses concerning key issues of the model. The simulations provide key information about the two knowledge trade-offs and enable to assess the systemic effects in terms of dynamic efficiency of alternative configurations of the intellectual property right regimes and architectures of the network interactions We have

explored the consequences of two sets of hypotheses: 1) the effects of different durations patents and 2) the effects of different architectural properties of the system in terms of distribution of firms with high levels of technological competence. Let us consider them in turn.

4.1. The first knowledge trade-off: Intellectual property right regimes

The first question the simulation has been employed to investigate refers to the role of patent protection in promoting and sustaining the innovation. The well-known IPR trade-off can now be investigated (Harison, 2008; Vandekerckhove and De Bond, 2008).

Intellectual property rights enable firms to secure exclusive rights on the technological knowledge they have generated. By means of IPR enterprises can exclude competitors from the exploitation of such new technologies and consolidate an effective competitive advantage. At the micro level patent protection reinforces the motivation to innovate giving the enterprise the possibility to exploit its own innovation in an exclusive way (hereafter “reinforcing effect”).

Moving from our basic assumption that the introduction of innovations builds upon in the recombination of existing knowledge it is clear that the patent protection has a negative effect: the longer the protection lasts the slower the new technologies can spread among enterprises (hereafter “slowing effect”) (Gay, Latham, Le Bas, 2008).

This research investigates both the effects focusing on the influence they have on the innovation process. The investigation has been based upon the exploitation of the simulation approach, based on the innovation model previously illustrated. The simulations has been run using the following model set up:

- all the firms (agents) operated into a common market and district,
- all the firms started from a similar level of technologies, randomly tossed into the first quarter of the achievable technologies following a uniform distribution,
- each firm was given high capability to observe the neighbours and to imitate public technologies,
- the unique parameter that varied among the simulations was the “patent expiration”, i. e. the time, in production cycles, a new technology was owned by the innovator and not available to the other agents in the system.

Two sets of experiments have been executed both based upon the observation of the average productivity level the agents achieved after a determined number of production cycles. In the model productivity is positively correlated to the technology, so the more a firm innovates the higher their productivity becomes: by observing the dynamics of the productivity it is possible to study the innovation strategy of the enterprises.

The first set of experiments consisted in benchmarking the innovation: this research studied the difference among the results (productivity level) obtained with several different duration of the patent protection and a benchmark figure, represented by the productivity level the agents achieved with patent expiration set to one.

To ensure the results were robust and systematic each simulation was run ten times by varying, for each run, the seed employed to generate pseudo random numbers; the result of each experiment was computed as the average of the ten runs results.

The second set of experiments consisted in correlating innovation and patent expiration: fifty simulations was run varying, each time, both the random seed and the value of the patent expiration parameter; the value was randomly tossed following a uniform distribution into the interval:]1,255[.

The described approach ensured both the robustness of the results and the independence of the parameters set up from any researcher's mental schemata.

4.1.1. Benchmarking the productivity

The following table 4.1.shows the average results obtained by running the simulation for ten times with a determined patent expiration and different random seeds. Each simulation was five hundred whole production cycles long. Data clearly show that the reinforcing effect is weaker than the slowing one, because the productivity level, directly dependent on the achieved technological level, decreases if the patent duration is higher⁴.

Results are resumed by the average of the distribution of values obtained from the simulations, the very low level of the variance ensure the significance of this average figure and suggests the results were systematic and fully independent by the random numbers distributions.

Patent expiration	Average productivity	Variance
1	0,2950635	6,16783E-05
8	0,2739246	2,54931E-05
55	0,234686	1,69686E-05
144	0,2003145	1,18582E-05
233	0,1835679	3,72715E-05

Table 4.1 average results of five sets of ten simulations with different patent duration and random seeds.

⁴ See Appendix A for the data.

The same results are reported into the figure 4.1 where each bar represents the average of the results obtained with a certain value for the parameter patent expiration. The graph shows that the four different scenarios (8, 55, 144, 233) were not able to achieve the benchmark, i. e. the slowing effect was constantly stronger than the reinforcing one.

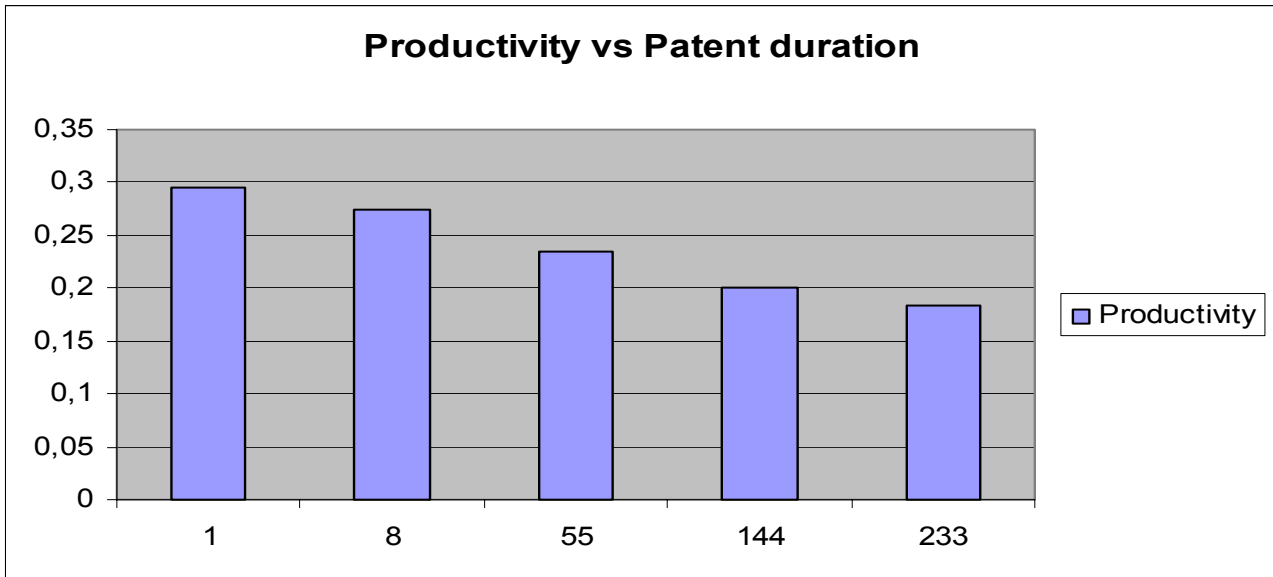


Figure 4.1 Histogram representing the results of the simulations

4.1.2 Correlating innovation and patent expiration

The results obtained by running fifty simulations, five hundred production cycles long, with random values for patent expiration demonstrate the existence of a negative correlation between patent rights and innovation: the longer the patent right is the less the productivity level grows, as graphically illustrated by the figure 4.1

The obtained correlation index is about -0.9; the distribution of the obtained results shows a remarkable relative difference between the best case (patent expiration = 6) and the worst one (patent expiration = 244). The two figures, respectively figure 4.2 and figure 4.3, illustrate the distribution of the average productivity values and the distributions of the relative difference between each value and the worst case one.

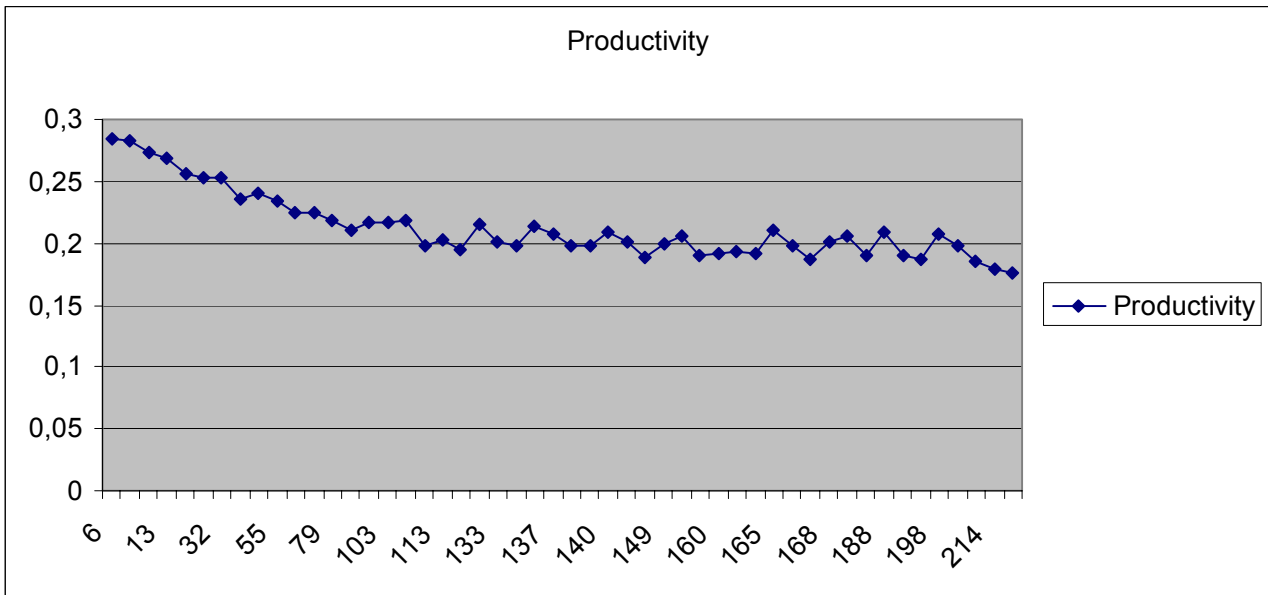


Figure 4.2 Distribution of the average productivity during fifty experiments with different settings of the patent expiration parameter

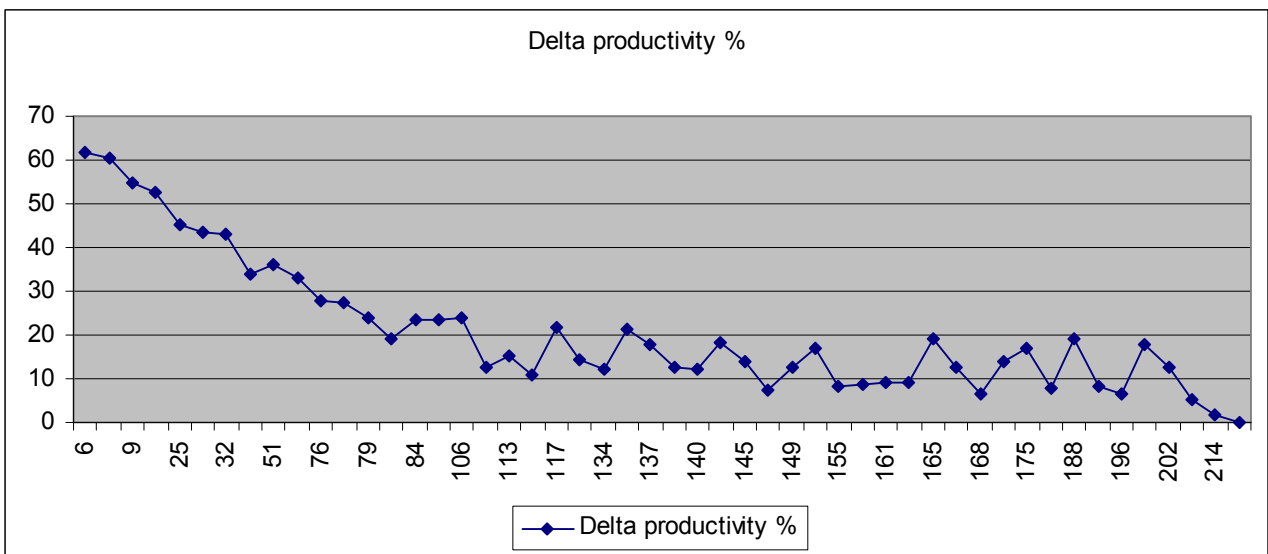


Figure 4.3 Distribution of the relative differences versus the worst case.

The relative difference (d) has been computed as:

$$4.1 \quad d_i = p_i / \min(p) * 100 - 100 .$$

Where p_i represents the productivity of the i -th experiments and $\min(p)$ the minimum productivity level achieved in all the experiments.

4.2. The second knowledge trade-off: the regional dissemination of knowledge

The second issue addressed by the simulation concerns to the role of the distribution in regional space of knowledge generating institutions, like research laboratories, universities and so on, in promoting and sustaining the innovation. We want to test the hypothesis that the dissemination of knowledge favours the growth of the system. This very first stage of the research has been focused on the influence that different architectural distributions of the knowledge producers could have on the dynamic of the innovation process.

The distribution in regional space of knowledge producers (hereafter KP) is a valuable source for the generation of new technological knowledge as they provide the opportunity to all the other co-localized agents to access part of their proprietary knowledge in the form of knowledge spillovers (Ozman, 2009).

In order to maintain the model at a useful level of simplicity, the knowledge producers have been dummied by some highly evolved firms whose distribution will affect the possibility for other firms to have imitate them.

The distribution of knowledge has been simulated by inserting a small number of enterprise endowed with a high level of technological knowledge (so called “genius”) into an environment populated by a wide set of less developed firms. The different distribution of genius and their number have been experimented in several scenarios, i.e. under diverse set up of some basic parameters that determine the quality of information available, the limits to the physical relocation, the capability to observe and copy others’ strategy and so on.

Four different distributions for knowledge producers have been studied and compared by observing the evolution of the productivity, directly dependent on the innovation rates, in four different spaces in which operated 250 normal enterprises and a certain number of knowledge-intensive firms. In each space the distribution of the high-tech enterprises was set up as follows:

- one high knowledge district (One hkd): all the KPs are placed, very close among them, in a small area at the centre of the space,
- two high knowledge districts (Two hkds): the total number of KPs is split between two areas, the first located at the centre of the right upper quarter of the space and the former at the centre of the left lower one,
- four high knowledge districts (Four hkds): here the KPs are distributed around four point, respectively at the centre of each quarter the whole space could be divided into,
- no high knowledge district (No hkds): each KP is assigned a random position into the space and lives alone.

Note that, whereas normal firms could move into the space, the KPs can not vary their position.

The basic population of each region (about 250 agents, due to the fact each agent is assigned a random space tossed following a uniform distribution) is randomly spread into the space.

Each set of experiments has been based upon a different combination of four parameters, so called scenario, each of them has been assigned a name:

- optimum: is the scenario devoted to re-create the theoretical condition of perfect information and mobility, there agents have a large view as well as knowledge is fully available and moving is always possible,
- typical: here the capabilities of the firms are limited to plausible amounts, in order to take in account the typical limits existing into the real world,
- mixed: the parameters have been randomly set up for each simulation, choosing their values into an assigned range that include the “typical” values.

For each scenario a set of three different experiments have been done, by using, respectively 4, 16 and 64 KPs for each space. By varying the number of KPs the difference between each KPs distribution model could be differently stressed: with 4 KPs for each space, there is few difference between the diverse distribution of them and, practically, the Four hkds and No hkds are equal. The more the number of KPs is increased the higher become the difference among the four distribution.

Scenario	Parameters				Number of CPs	Experiment
	View	Jump	Imitation Threshold	Patent Expiration		
Optimum	15	50	999	1	4	Optimum4
					16	Optimum16
					64	Optimum64
Typical	4	4	4	5	4	Typical4
					16	Typical16
					64	Typical64
Mixed]0,8[]0,8[]1,9[]1,15[4	Mixed4
					16	Mixed16
					64	Mixed64

Table 4.2 – Parameter configurations for each experiment.

Each experiment has been repeated for fifty times always changing the random number distribution to simulate different dynamics and validate the robustness of the obtained results. Random numbers were used to simulate some decision, to pick up neighbours to imitate and to determine in which direction and how far to move. For the Mixed scenario random numbers are used to toss the parameters value each

within the appropriate range, as illustrated in Table 4.2 where parameters for each scenario and simulation are shown⁵.

At the end of each experiment the average productivity level for each region and for the whole population have been computed, at this very first stage of the research these were the only data it has been decided to concentrate on. Since the initial endowment of the firms in each region was set to the same amount, the market was unique both for factors and products, it is possible to assume differences among the reached level of productivity were mainly due to the different distribution of the KPs; the figure 4.4 shows this distribution, KPs are drawn in violet.

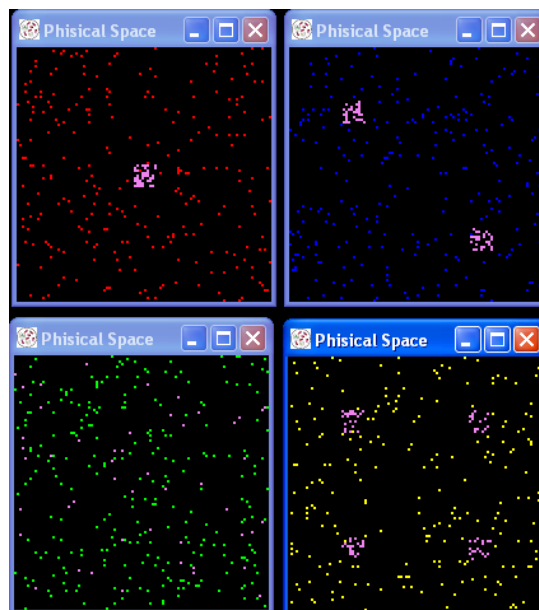


Figure 4.4 - Configuration of the spaces for the simulation.

4.2.1 Results of the Optimum scenario

The “Optimum” scenario has been set up to validate the model under the classic assumption of perfect information and mobility: provided that each physical space is simulated by a square lattice one hundred cells wide, jumping in each direction of fifty cells means have a perfect mobility, as well as because the maximum distance between the worst and the best technology is 200 an imitation threshold of 999 means that each technology could be copied.

The patent expiration set to one means that each adopted technology becomes public in the successive production cycle, so each technology could be copied as soon as it has been adopted.

⁵ See Appendix B for the data.

The value of the view parameter would have been set to fifty too, as for the jump one, but fifteen demonstrated to be enough to allow a good circulation of information and guarantee the majority of the enterprises reached the higher technological level in very few time.

The figure 4.5 shows the dynamic of the productivity during a run of this scenario, there it is evident the whole population rapidly reach the maximum productivity level, set up by construction of the model to one.

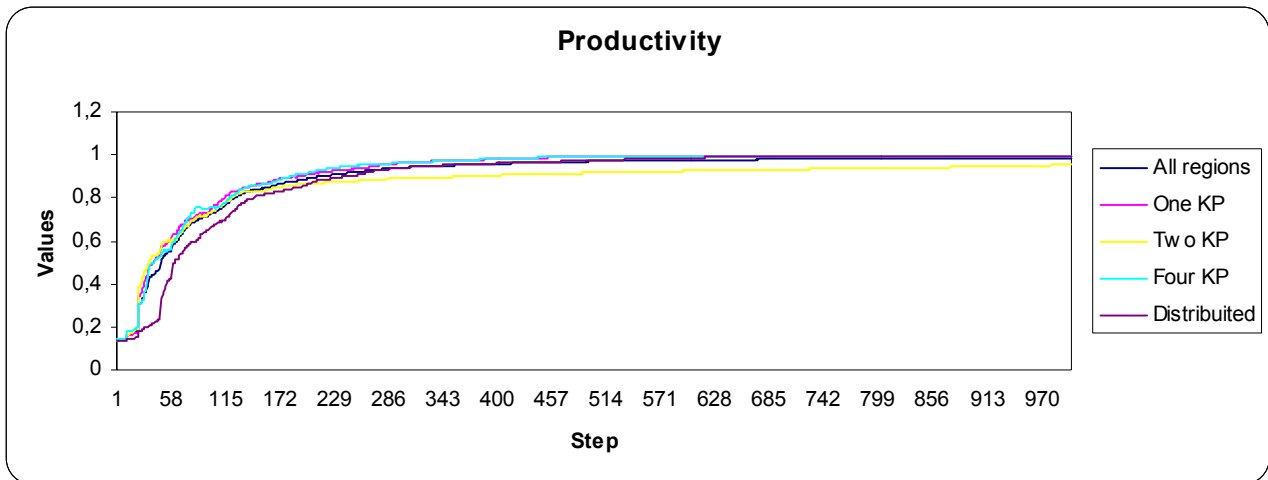


Figure 4.5 - Evolution of the average productivity in the Optimum scenario

Under the optimum conditions, the concentrated distributions of KPs, as the One hkd and Two hkds seem to give some advantages, as shown by the results briefly summarized into the table 4.3. Here are reported, for each experiment, the average results, first row, obtained during fifty runs, with different random distributions, each of them 250 whole production cycles long, the variance is reported too, in the second row.

Experiment		All	One hkd	Two hkds	Four hkds	No hkds
Optimum 4	Average	0,92933926	0,95978226	0,9399946	0,87545882	0,85853482
	Variance	0,001314552	0,00163349	0,013786755	0,031748939	0,026233417
Optimum 16	Average	0,98579554	0,99177854	0,99315214	0,98425224	0,97135544
	Variance	0,00010812	7,43194E-05	1,9776E-05	0,000629735	0,001899397
Optimum 64	Average	0,9944335	0,99499734	0,99496726	0,99445056	0,99328388
	Variance	1,23378E-06	1,92086E-11	1,05831E-08	4,31916E-07	2,24736E-05

Table 4.3 – Synthesis of the results obtained by running the Optimum scenario.

Under high availability of information, perfect mobility, absolute capability of each enterprise to perfectly imitate each other, and no patent protections the concentrate model for knowledge centres seem to give better results than them more spread, even if the advantage becomes smaller and smaller when the number of KPs grows: with only 4 KPs the spread region reached only 0.85 productivity after 250 production

cycles, whereas the full concentrated one reached 0.95, with an advantage of about 0.1, but this difference fell to 0.02 and 0.001 respectively with 16 and 64 KPs. The detailed results of each run are available in Appendix A.

4.2.2 Results of the Typical scenario

This configuration set has been obtained by giving the four parameters realistic and plausible values, the physical neighbourhood of each firm has been presumed to be 64 cells wide, about 1/100 the whole extension of the simulated world, where each cell was able to host more than one enterprise. Pretending this neighbourhood to be the maximum extension an enterprise would have been able to reach, the possibility to move has been limited at the same amount.

Innovation can not be done too fast, adopting new technologies would mean modify processes and upgrade the staff skills, so it is not plausible that an enterprise can imitate whatever other ones even they are far away on technologies. The limit of 4, represents 1/50 of the maximum technology an enterprise can reach in the whole evolution, and four hundred times the ability each enterprise is pretended to acquire each cycle by means of the “learning by doing”.

It is also plausible that new techniques could be protected by a license, usually technical patents last for five years, because each step of the simulation is pretended to last for one year, the expiration of patent rights has been set up to five. Practically each enterprise could observe and imitate the other ones technologies only if they are five cycles old.

All these limitations reduced the speed of evolution, so experiments for this scenario has been based upon one thousand cycles simulations long, even though the enterprises reached productivity levels less than them obtained in the, non realistic, Optimum scenario. The interesting results is that, under more realistic conditions relevant indications about the better distribution of KPs seem to appear; as in the table 4.4, where are shown the average results of fifty runs for each experiment using the Typical scenario.

Experiment		All	One hkd	Two hkds	Four hkds	No hkds
Typical 4	Average	0,4949331	0,45518152	0,4803268	0,50164072	0,50874628
	Variance	0,000444118	0,005142019	0,007908483	0,00884733	0,007251393
Typical 16	Average	0,72644622	0,65639706	0,69547358	0,73852646	0,78994042
	Variance	0,000390484	0,003172301	0,001372866	0,001981927	0,00180454
Typical 64	Average	0,88363828	0,796893	0,84444666	0,90662872	0,95715874
	Variance	9,09408E-05	0,001474684	0,000629902	0,000102324	5,85965E-05

Table 4.4 - Synthesis of the results obtained by running the Typical scenario.

In all the three set up of number of KPs the spread model supplies better results, and the distance become higher the higher the number of KPs is. Analysing the four

regions it is evident that the more the KPs are spread the better become the results, the advantage grows significantly passing from the One hkd region to the No hkds ones, reaching, for 64 KPs, 0,16.

More spread distributions of the KPs seem to be more effective in facilitating the innovation and in promote technical progress that rise the productivity of the firms. A plausible explanation could be that more spread distributions allow a major number of enterprises to access knowledge, bypassing difficulties due to the limited capability, and time too, the enterprises experiment in the real world; such a conclusion seems to be confirmed by the value, very close, for Four hkds and No hkds in presence of four only KPs, when the distributions of KPs are more or less equivalent.

4.2.3. Results of the Mixed scenario

The Mixed scenario has been built to test the results obtained into the typical one, here the parameters set up is always changing, values are randomly tossed in ranges that are distributed around the typical parameters value.

The results, reported in table 4.5, confirm them obtained by running the typical scenario, so the previous reasoning about the importance of a spread model for KPs seems to be reinforced, as well as the observation about the similarity between the distribution Four hkds and No hkds in presence of four KPs only.

The difference among the four distribution is less strong, due to the fact the combination of parameters allowed configurations closer to the Optimum scenario than the Typical ones.

Experiment		All	One hkd	Two hkds	Four hkds	No hkds
Mixed 4	Average	0,47004768	0,44459296	0,45632052	0,47657632	0,47455614
	Variance	0,019376728	0,023025456	0,025161962	0,026423885	0,024439737
Mixed 16	Average	0,6359485	0,57382926	0,59963624	0,64910046	0,69728222
	Variance	0,017227812	0,023181741	0,018040034	0,019603033	0,018044497
Mixed 64	Average	0,83311512	0,771741	0,79950118	0,84245002	0,89523222
	Variance	0,016237806	0,023435189	0,01945329	0,017575774	0,01435437

Table 4.5 - Synthesis of the results obtained by running the Mixed scenario.

In sum, taking advantage of the array of experimental configurations that agent based simulations offer, we have generated a wide set of alternative scenarios. For a comparative summarization it is possible to refer to the graph in figure 4.6 where the three table shown before are mixed in a bar diagram. The advantage of the “No hkds” distribution is clear in the scenario Typical and Mixed, whereas in the Optimum scenario results are very similar for the three different distributions.

The bar diagram shows also the performance of the spread distributions are better the higher is the number of KPs, reinforcing the previous reasoning about the similarity of distributions in presence of few KPs.

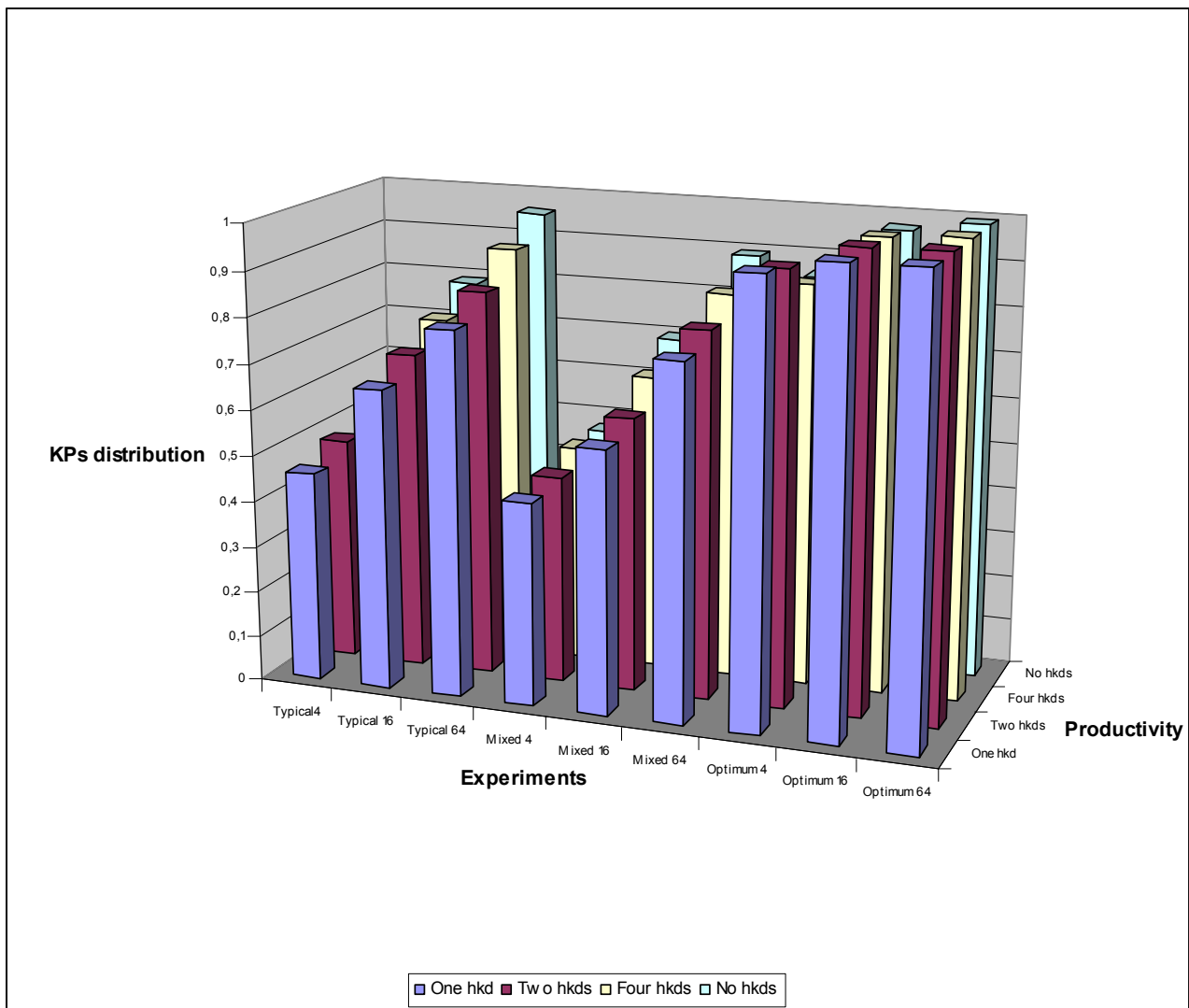


Figure 4.6 – Comparison among results obtained during the different experiments

5. Conclusion

According to a well-established tradition of economic analysis, founded by Alfred Marshall and Simon Kuznetz the structural characteristics of the economic system into which firms are embedded play a crucial role to assessing their performances. In this context the Arrovian notion of knowledge indivisibility enables to explore the effects that alternative institutional and architectural configurations of the knowledge structure play in assessing the chances to generate new technological knowledge and to introduce technological innovations. The reappraisal of the Schumpeterian notion of innovation as a conditional result of a form of reaction to un-expected events, leads to articulate the hypothesis that the reaction of myopic but creative agents, that try

and cope with the changing conditions of their product and factor markets, may lead to the introduction of productivity enhancing innovations when they can actually take advantage of the external knowledge available within the innovation system.

In our approach external knowledge is an indispensable input, together with internal research activities, into the generation of new knowledge. The introduction of innovations is analyzed as the result of systemic interactions among learning agents. The creative reaction of agents may lead to the introduction of productivity enhancing innovations. This takes place only when they can actually take advantage of external knowledge available within the innovation system into which they are embedded. Building upon agent-based simulation techniques the paper explores the effects that alternative configurations of the intellectual property right regimes and architectural configurations play in assessing the chances to generate new technological knowledge. The aim of the paper was to identify and disentangle the two knowledge dissemination trade-offs between alternative institutional, geographical and architectural settings of the structure into which knowledge interactions take place.

The results of the agent-based simulations confirm that a system characterized by high levels of knowledge dissemination is actually more effective in promoting the rates of introduction of technological innovations. The results however show that systems characterized by high levels of concentration could offer advantages in terms of faster discovery, due to the close relations that could be established among the knowledge producers.

The implementation of an agent-based simulation model has enabled the rigorous framing of a complex system dynamics where innovation is the emerging property that takes place when a number of complementary conditions qualify the reaction of firms and make them creative. The simulation model can be applied to control the implications of an array of alternative settings and hypotheses concerning appropriability conditions, intellectual property rights regimes, and knowledge generation routines.

Summarizing the results of the present simulation it seems to be possible to pretend that the more the knowledge producers, like universities, are spread upon the territory and the faster and more effective becomes the innovation process. Myopic but creative firms coping with the changing conditions of their product and factor markets are better able to improve their reaction and make is creative, as opposed to adaptive, when technological knowledge is disseminated in the regional, institutional and technological space.

The implications for research and economic policy are important: better access conditions to technological knowledge enable firms to find better their way toward technological enhancement so as to become more competitive and profitable.

The access to technological knowledge should be increased favouring the distribution of universities and public research centres across the state so as to improve the proximity of firms to the available pools of public knowledge and reduce the distance of peripheral regions from the knowledge spillovers. In a similar vein the intellectual property rights regimes should be designed so as to increase the possibility for imitators and users of external knowledge to take advantage of existing proprietary knowledge. The implementation of non-exclusive intellectual property rights might favour the dissemination of technological knowledge. The enforcement of compulsory royalty payments for all use of proprietary knowledge should prevent the reduction of appropriability conditions and hence the decline of incentives to funding research activities. Finally, the demise of ‘intramuros’ research activities concentrated within the research laboratories of large corporations and the implementation of open innovation systems that favour the outsourcing of the generation of technological knowledge to specialized knowledge-intensive business companies, and academic departments might help the dissemination of technological knowledge.

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Appendix A – Data of the correlation experiment

Patent duration	Productivity	Delta %	Delta
6	0,284921	61,779385	0,06
7	0,282326	60,305933	0,07
9	0,272979	54,998666	0,09
13	0,268845	52,651362	0,13
25	0,255922	45,313627	0,25
32	0,252791	43,535831	0,32
32	0,252286	43,24909	0,32
49	0,235599	33,774139	0,49
51	0,239775	36,14529	0,51
55	0,234053	32,896313	0,55
76	0,224799	27,641852	0,76
77	0,224557	27,504443	0,77
79	0,21831	23,957369	0,79
82	0,209897	19,180431	0,82
84	0,217206	23,330513	0,84
103	0,21712	23,281682	1,03
106	0,218301	23,952259	1,06
108	0,198477	12,696105	1,08
113	0,203079	15,309141	1,13
116	0,195488	10,998938	1,16
117	0,214648	21,87807	1,17
133	0,201319	14,309805	1,33
134	0,19746	12,118648	1,34
135	0,213417	21,179103	1,35
137	0,207711	17,939211	1,37
138	0,198245	12,564375	1,38
140	0,197665	12,235048	1,4
140	0,208514	18,395158	1,4
145	0,200829	14,031581	1,45
147	0,18891	7,2639211	1,47
149	0,198693	12,818751	1,49
152	0,205999	16,96713	1,52
155	0,19078	8,3257153	1,55
160	0,191412	8,6845676	1,6
161	0,192426	9,2603213	1,61
163	0,192133	9,0939546	1,63
165	0,209841	19,148634	1,65
166	0,19856	12,743233	1,66
168	0,18749	6,457639	1,68
168	0,200949	14,099718	1,68
175	0,205845	16,879688	1,75
180	0,189769	7,7516651	1,8
188	0,209592	19,007251	1,88
189	0,190723	8,2933504	1,89
196	0,187485	6,4547999	1,96
198	0,207803	17,991449	1,98
202	0,198069	12,464441	2,02
207	0,185588	5,3776751	2,07
214	0,178835	1,5432922	2,14
247	0,176117	0	2,47

Appendix B – Detailed results of the experiments

Experiment Typical 4					
Execution	Results				
	All	One hkd	Two hkds	Four hkds	No hkds
1	0,508894	0,471829	0,456458	0,471337	0,61737
2	0,521951	0,455364	0,510993	0,434025	0,65883
3	0,492962	0,389795	0,598831	0,556899	0,370867
4	0,47801	0,412681	0,478719	0,555686	0,447857
5	0,459548	0,417952	0,512897	0,4435	0,456089
6	0,488209	0,519965	0,325125	0,427917	0,648381
7	0,482839	0,544783	0,587778	0,46625	0,19878
8	0,484841	0,528984	0,390684	0,509539	0,485069
9	0,512594	0,494918	0,456262	0,562782	0,521326
10	0,48239	0,360143	0,572321	0,298297	0,60646
11	0,489271	0,515593	0,493781	0,528239	0,400866
12	0,476028	0,386132	0,446991	0,323967	0,65351
13	0,487608	0,405667	0,559835	0,489825	0,493881
14	0,520747	0,542126	0,644188	0,329396	0,487577
15	0,545019	0,641565	0,555675	0,521271	0,44316
16	0,484554	0,436341	0,524159	0,366814	0,589783
17	0,512651	0,441818	0,458917	0,595201	0,518374
18	0,465467	0,395337	0,300594	0,641577	0,448957
19	0,511568	0,459779	0,479478	0,490042	0,609355
20	0,484686	0,533929	0,393024	0,554216	0,45144
21	0,514686	0,50344	0,534843	0,543	0,468664
22	0,533085	0,493542	0,523508	0,595729	0,509147
23	0,494312	0,478823	0,483	0,448941	0,556752
24	0,503018	0,467523	0,31848	0,562007	0,597114
25	0,462775	0,34801	0,4245	0,494715	0,536121
26	0,479584	0,452481	0,354872	0,648014	0,430826
27	0,48846	0,482261	0,428814	0,548204	0,48192
28	0,499232	0,49244	0,585241	0,32981	0,548174
29	0,48499	0,496215	0,510688	0,47552	0,456929
30	0,511187	0,475853	0,316716	0,503	0,663144
31	0,522364	0,60375	0,456684	0,526737	0,479407
32	0,479758	0,383387	0,462752	0,439667	0,612206
33	0,489605	0,412909	0,517446	0,520435	0,498333
34	0,472521	0,497455	0,444264	0,428992	0,524587
35	0,436677	0,437187	0,387985	0,462026	0,470191
36	0,525144	0,536557	0,424958	0,622695	0,488691
37	0,503022	0,482087	0,45807	0,610308	0,438538
38	0,51864	0,367545	0,615368	0,531897	0,531821
39	0,495404	0,505403	0,5394	0,346386	0,5555
40	0,485847	0,424196	0,492689	0,512143	0,504962
41	0,505645	0,420044	0,571276	0,563633	0,435051
42	0,524932	0,551461	0,579356	0,577963	0,366071
43	0,478492	0,32319	0,491229	0,578311	0,494018
44	0,464919	0,317217	0,561807	0,371336	0,581429
45	0,500673	0,489258	0,633722	0,311264	0,507534
46	0,496185	0,451865	0,446845	0,582635	0,4749
47	0,519509	0,326989	0,497717	0,632791	0,537024
48	0,493692	0,3307	0,385893	0,63305	0,546356
49	0,494296	0,446606	0,549667	0,507403	0,467712
50	0,478164	0,405981	0,27184	0,606644	0,56626
Maximum	0,545019	0,641565	0,644188	0,648014	0,663144
Average	0,4949331	0,45518152	0,4803268	0,50164072	0,50874628
Variance	0,00044412	0,00514202	0,00790848	0,00884733	0,00725139

Experiment Typical 16					
Execution	Results				
	All	One hkd	Two hks	Four hks	No hks
1	0,743919	0,71743	0,765928	0,745562	0,745029
2	0,715707	0,724722	0,698275	0,689206	0,748634
3	0,707172	0,618421	0,700306	0,713956	0,778936
4	0,714789	0,627794	0,637284	0,78648	0,76862
5	0,720443	0,726222	0,641329	0,726006	0,779497
6	0,697783	0,587171	0,678667	0,726538	0,763676
7	0,732966	0,605	0,721118	0,777755	0,792965
8	0,718619	0,652883	0,68552	0,727613	0,799516
9	0,719606	0,620923	0,636324	0,743152	0,83483
10	0,722214	0,621222	0,690647	0,774359	0,770742
11	0,710598	0,625322	0,665359	0,808874	0,732853
12	0,759372	0,744342	0,741951	0,76173	0,783015
13	0,749283	0,604133	0,755829	0,787228	0,812606
14	0,725296	0,690549	0,644355	0,816821	0,729135
15	0,780771	0,719556	0,691006	0,787189	0,884742
16	0,718642	0,635	0,688081	0,714364	0,812398
17	0,760266	0,746135	0,691845	0,798564	0,790272
18	0,731544	0,623648	0,728042	0,793031	0,764972
19	0,742874	0,713223	0,706073	0,667089	0,859116
20	0,711768	0,587548	0,659223	0,746105	0,819171
21	0,721227	0,669939	0,668283	0,678224	0,829612
22	0,726192	0,691892	0,727045	0,771609	0,71676
23	0,739599	0,709769	0,718687	0,668097	0,830493
24	0,699883	0,58478	0,642484	0,75307	0,794622
25	0,716184	0,674588	0,759952	0,599231	0,78359
26	0,716161	0,705947	0,703893	0,762344	0,684157
27	0,747319	0,729395	0,689784	0,747278	0,805697
28	0,743032	0,679118	0,760631	0,676087	0,826728
29	0,703676	0,528929	0,748711	0,684026	0,793238
30	0,719573	0,669185	0,697364	0,683464	0,81317
31	0,704347	0,568896	0,715303	0,754972	0,758913
32	0,739916	0,649085	0,706087	0,746793	0,822844
33	0,710591	0,643903	0,62292	0,729225	0,809391
34	0,762052	0,699878	0,729806	0,772527	0,830267
35	0,720284	0,656311	0,67997	0,71227	0,814692
36	0,674187	0,550385	0,683478	0,747446	0,690124
37	0,726688	0,735242	0,655722	0,758249	0,761907
38	0,706321	0,65513	0,664936	0,744976	0,747197
39	0,702168	0,689252	0,678619	0,68741	0,750172
40	0,743392	0,577236	0,676921	0,768143	0,866698
41	0,755511	0,710539	0,684167	0,781754	0,816336
42	0,730088	0,664062	0,721704	0,709714	0,80873
43	0,717606	0,588185	0,744925	0,720452	0,780598
44	0,719863	0,610033	0,727722	0,705599	0,808959
45	0,750063	0,772553	0,653572	0,731118	0,82074
46	0,728125	0,68825	0,636455	0,807943	0,754059
47	0,71434	0,626344	0,678161	0,772297	0,765862
48	0,743175	0,632555	0,740829	0,705225	0,841815
49	0,733343	0,633307	0,713922	0,694454	0,842417
50	0,723773	0,633921	0,714464	0,760704	0,756508
Maximum	0,780771	0,772553	0,765928	0,816821	0,884742
Average	0,72644622	0,65639706	0,69547358	0,73852646	0,78994042
Variance	0,00039048	0,0031723	0,00137287	0,00198193	0,00180454

Experiment Typical 64					
Execution	Results				
	All	One hkd	Two hks	Four hks	No hks
1	0,871796	0,765022	0,842387	0,905	0,947925
2	0,874	0,804335	0,839831	0,892584	0,945438
3	0,883985	0,809083	0,82094	0,910135	0,963389
4	0,886406	0,77927	0,875765	0,908874	0,957118
5	0,892403	0,83141	0,865918	0,898973	0,957467
6	0,904144	0,86	0,854068	0,919672	0,967163
7	0,886876	0,788549	0,851322	0,912606	0,961238
8	0,875836	0,78068	0,842984	0,907879	0,946216
9	0,884772	0,801684	0,852939	0,899843	0,955103
10	0,878662	0,777795	0,833092	0,904226	0,962297
11	0,874877	0,76118	0,852537	0,906793	0,952074
12	0,901325	0,844854	0,879577	0,907703	0,954661
13	0,871881	0,730924	0,831175	0,902977	0,961865
14	0,881499	0,777643	0,83437	0,925115	0,950877
15	0,874438	0,812606	0,785365	0,895801	0,968077
16	0,899017	0,84344	0,854648	0,912967	0,9625
17	0,875467	0,720433	0,843353	0,912768	0,961765
18	0,875341	0,73518	0,878432	0,900137	0,957863
19	0,890298	0,830661	0,886364	0,900202	0,935565
20	0,879648	0,793341	0,819727	0,91244	0,956853
21	0,886887	0,787895	0,872023	0,909877	0,951716
22	0,881121	0,804244	0,843934	0,898472	0,960494
23	0,870098	0,739434	0,851071	0,907918	0,949909
24	0,87917	0,817275	0,829123	0,89801	0,958286
25	0,876167	0,790378	0,824024	0,898675	0,959805
26	0,900714	0,845482	0,854113	0,921013	0,970144
27	0,891328	0,843416	0,838975	0,911847	0,955194
28	0,879885	0,728864	0,842543	0,910151	0,967186
29	0,878119	0,782256	0,830983	0,926395	0,953375
30	0,88322	0,803133	0,827222	0,905899	0,965662
31	0,88847	0,792977	0,854004	0,918193	0,955799
32	0,896761	0,854344	0,856422	0,913384	0,950261
33	0,888726	0,81037	0,845322	0,910281	0,963181
34	0,895037	0,866802	0,850082	0,901147	0,950386
35	0,869292	0,750283	0,826567	0,9044	0,959431
36	0,90186	0,848593	0,880885	0,917099	0,955691
37	0,873235	0,814303	0,821102	0,903359	0,94134
38	0,87879	0,784451	0,848793	0,911245	0,949071
39	0,875134	0,819053	0,7865	0,921724	0,953622
40	0,887834	0,82976	0,830642	0,921426	0,95559
41	0,885372	0,813951	0,834958	0,900286	0,954126
42	0,880257	0,803186	0,848163	0,884693	0,958079
43	0,87855	0,686018	0,869767	0,898305	0,968266
44	0,881903	0,816055	0,831852	0,896591	0,953109
45	0,885155	0,800968	0,894004	0,87971	0,949616
46	0,886651	0,78391	0,864447	0,899623	0,962194
47	0,86429	0,807562	0,76	0,888408	0,954383
48	0,887042	0,761005	0,854959	0,913455	0,967215
49	0,886035	0,801025	0,835419	0,900459	0,973422
50	0,90214	0,809567	0,86964	0,922696	0,96593
Maximum	0,904144	0,866802	0,894004	0,926395	0,973422
Average	0,88363828	0,796893	0,84444666	0,90662872	0,95715874
Variance	9,0941E-05	0,00147468	0,0006299	0,00010232	5,8597E-05

Experiment MIXED 4					
Execution	Results				
	All	One hkd	Two hkds	Four hkds	No hkds
1	0,267171	0,240139	0,308186	0,318611	0,198283
2	0,63339	0,591617	0,53531	0,775366	0,566903
3	0,599828	0,641165	0,697695	0,537941	0,509921
4	0,542666	0,528361	0,450603	0,619583	0,558662
5	0,540789	0,54801	0,539975	0,539394	0,535305
6	0,454002	0,484587	0,5176	0,51496	0,276651
7	0,670008	0,676844	0,480151	0,745988	0,738509
8	0,275412	0,256084	0,25875	0,324388	0,257143
9	0,486988	0,525748	0,517746	0,393077	0,495074
10	0,470136	0,400342	0,526037	0,487746	0,455571
11	0,279389	0,213462	0,233119	0,307864	0,349958
12	0,639777	0,665221	0,575305	0,445656	0,802255
13	0,436975	0,331978	0,43636	0,446814	0,500368
14	0,321166	0,321413	0,355825	0,374674	0,235152
15	0,21007	0,222149	0,20848	0,1889	0,218053
16	0,588752	0,477923	0,577956	0,61964	0,663548
17	0,208555	0,208789	0,205165	0,215972	0,203247
18	0,616849	0,666503	0,620311	0,566142	0,603347
19	0,227249	0,177443	0,214563	0,226848	0,293837
20	0,504949	0,585303	0,557801	0,223595	0,561707
21	0,623512	0,534628	0,720148	0,57684	0,631458
22	0,591202	0,252011	0,658481	0,69089	0,608279
23	0,384881	0,43042	0,25489	0,517539	0,285625
24	0,535847	0,630326	0,537557	0,583074	0,367083
25	0,542299	0,544224	0,540239	0,538857	0,545949
26	0,588653	0,571225	0,592518	0,60745	0,583321
27	0,493694	0,44193	0,489188	0,530185	0,50543
28	0,170068	0,183317	0,157703	0,139101	0,193
29	0,544823	0,490658	0,606125	0,382957	0,653781
30	0,589725	0,674767	0,515581	0,522269	0,622619
31	0,446961	0,468516	0,373	0,365429	0,544007
32	0,283966	0,27017	0,287165	0,2857	0,291495
33	0,588209	0,492056	0,593771	0,738114	0,452131
34	0,193215	0,177926	0,182273	0,196962	0,213066
35	0,53693	0,488823	0,520833	0,621	0,496976
36	0,570777	0,546306	0,723202	0,419711	0,52689
37	0,518791	0,413632	0,529181	0,603365	0,503382
38	0,297302	0,277035	0,282019	0,231966	0,378973
39	0,440138	0,46645	0,540179	0,365675	0,329589
40	0,487788	0,498333	0,201099	0,568819	0,589014
41	0,437198	0,296667	0,504104	0,4875	0,439065
42	0,611782	0,528893	0,648804	0,640099	0,615814
43	0,495788	0,512083	0,446765	0,475318	0,53816
44	0,557597	0,59451	0,210102	0,686	0,612993
45	0,284486	0,215857	0,301184	0,324633	0,298807
46	0,500714	0,434125	0,494267	0,526825	0,538116
47	0,520456	0,507047	0,590375	0,477117	0,515176
48	0,493	0,370934	0,39432	0,578095	0,585181
49	0,62387	0,601828	0,560175	0,707368	0,609073
50	0,574591	0,55187	0,54384	0,566799	0,62986
Maximum	0,670008	0,676844	0,723202	0,775366	0,802255
Average	0,47004768	0,44459296	0,45632052	0,47657632	0,47455614
Variance	0,01937673	0,02302546	0,02516196	0,02642389	0,02443974

Experiment MIXED 16					
Execution	Results				
	All	One hkd	Two hkds	Four hkds	No hkds
1	0,431633	0,350385	0,386809	0,420454	0,555641
2	0,83369	0,825651	0,750518	0,846127	0,900837
3	0,81609	0,754898	0,688191	0,836776	0,921765
4	0,563121	0,528139	0,578699	0,568699	0,576021
5	0,735944	0,653066	0,704842	0,745698	0,804364
6	0,688221	0,651618	0,711609	0,695063	0,687198
7	0,712717	0,726037	0,611824	0,701604	0,791231
8	0,471252	0,372182	0,490169	0,537169	0,464535
9	0,720947	0,673	0,731324	0,730542	0,745408
10	0,552207	0,557544	0,545341	0,538322	0,567628
11	0,725853	0,578345	0,757949	0,731586	0,80116
12	0,578288	0,448504	0,493967	0,630469	0,708299
13	0,684953	0,633915	0,720268	0,6715	0,701063
14	0,782275	0,835239	0,702982	0,766908	0,814572
15	0,400472	0,27492	0,30265	0,402836	0,5682
16	0,683358	0,582326	0,694849	0,624245	0,79301
17	0,625211	0,483806	0,541619	0,628456	0,781088
18	0,572478	0,430174	0,503185	0,595625	0,697122
19	0,542145	0,534622	0,546771	0,544158	0,543708
20	0,469421	0,369202	0,348044	0,467046	0,629156
21	0,559429	0,579968	0,545273	0,570234	0,540676
22	0,437186	0,347324	0,432828	0,424677	0,523392
23	0,54886	0,54655	0,550311	0,552174	0,546476
24	0,779881	0,795707	0,775641	0,783184	0,762719
25	0,774239	0,701206	0,75627	0,814162	0,815308
26	0,615824	0,535294	0,584901	0,639172	0,6948
27	0,631797	0,47854	0,640753	0,593007	0,772293
28	0,554153	0,57989	0,548389	0,548097	0,540699
29	0,83449	0,834347	0,809089	0,806973	0,88171
30	0,800681	0,799922	0,806138	0,738869	0,843273
31	0,807591	0,694006	0,743083	0,873198	0,873744
32	0,575498	0,549366	0,492868	0,616739	0,638333
33	0,70373	0,552712	0,616895	0,789508	0,774707
34	0,561191	0,56056	0,580466	0,548678	0,55565
35	0,56147	0,561689	0,550844	0,563448	0,570371
36	0,478563	0,414634	0,419031	0,500454	0,552322
37	0,450705	0,36675	0,463617	0,447692	0,518162
38	0,498577	0,440413	0,405139	0,520263	0,592532
39	0,786468	0,781454	0,726319	0,789333	0,839603
40	0,569009	0,447244	0,572831	0,586364	0,649935
41	0,624603	0,506418	0,58426	0,776043	0,576024
42	0,749205	0,571536	0,662826	0,821345	0,869196
43	0,871295	0,853199	0,801738	0,905697	0,907406
44	0,656294	0,533678	0,575	0,710975	0,764487
45	0,824708	0,794267	0,744789	0,843982	0,898482
46	0,408535	0,337196	0,333539	0,368369	0,540795
47	0,593613	0,417153	0,533729	0,623103	0,770975
48	0,555024	0,555523	0,542139	0,555871	0,568965
49	0,556511	0,545446	0,556222	0,574701	0,551423
50	0,838019	0,745898	0,815274	0,885428	0,877647
Maximum	0,871295	0,853199	0,815274	0,905697	0,921765
Average	0,6359485	0,57382926	0,59963624	0,64910046	0,69728222
Variance	0,01722781	0,02318174	0,01804003	0,01960303	0,0180445

Experiment MIXED 64					
Execution	Results				
	All	One hkd	Two hkds	Four hkds	No hkds
1	0,651627	0,490709	0,559188	0,669602	0,807428
2	0,760087	0,605436	0,640382	0,775628	0,925807
3	0,852494	0,771818	0,831592	0,862586	0,921798
4	0,501632	0,501499	0,501672	0,501619	0,501757
5	0,974778	0,956407	0,966599	0,987358	0,988337
6	0,939814	0,929763	0,925016	0,949064	0,955431
7	0,745134	0,67861	0,712404	0,732818	0,831146
8	0,955499	0,946877	0,945121	0,952246	0,975939
9	0,507516	0,511474	0,505085	0,504672	0,508378
10	0,88146	0,7782	0,820506	0,924028	0,96952
11	0,856929	0,812085	0,82953	0,858235	0,915429
12	0,815461	0,705127	0,774523	0,840332	0,902206
13	0,888845	0,827325	0,844703	0,901078	0,970165
14	0,918516	0,86653	0,896966	0,937464	0,961309
15	0,809418	0,701682	0,733738	0,814024	0,941947
16	0,938263	0,923057	0,924852	0,93795	0,965837
17	0,990371	0,993576	0,992853	0,988167	0,986814
18	0,716397	0,576695	0,683102	0,751442	0,819282
19	0,914689	0,825848	0,895585	0,938344	0,970591
20	0,697014	0,623115	0,631913	0,723301	0,797826
21	0,750188	0,653208	0,720765	0,757722	0,840923
22	0,798157	0,787537	0,815278	0,798019	0,794388
23	0,709717	0,65679	0,648791	0,732854	0,7893
24	0,524419	0,519936	0,523861	0,531347	0,522261
25	0,943604	0,933804	0,938415	0,943147	0,957855
26	0,934291	0,93	0,917836	0,940065	0,948292
27	0,917562	0,87448	0,88225	0,92918	0,972442
28	0,665879	0,562837	0,5525	0,656222	0,834555
29	0,81042	0,652254	0,770027	0,803097	0,933436
30	0,878345	0,73068	0,8543	0,904231	0,966656
31	0,965624	0,958781	0,932525	0,979856	0,987127
32	0,870775	0,768266	0,846697	0,902977	0,945702
33	0,938343	0,939787	0,934839	0,936912	0,939912
34	0,932392	0,889264	0,901731	0,948999	0,9782
35	0,754796	0,681853	0,74367	0,751287	0,834799
36	0,9144	0,853486	0,891738	0,925943	0,973475
37	0,665056	0,513876	0,566455	0,566585	0,889231
38	0,97771	0,98416	0,957864	0,979158	0,988081
39	0,831373	0,73689	0,81989	0,856929	0,891088
40	0,939966	0,914795	0,916936	0,945494	0,979749
41	0,713078	0,70137	0,730662	0,720068	0,703427
42	0,979879	0,97737	0,967708	0,986796	0,986283
43	0,811996	0,612979	0,713144	0,864901	0,929835
44	0,716139	0,651613	0,658851	0,725322	0,804364
45	0,764351	0,616984	0,680631	0,781935	0,919523
46	0,946769	0,92227	0,89574	0,971818	0,986954
47	0,959257	0,966013	0,957609	0,958889	0,953051
48	0,852558	0,763116	0,808586	0,874937	0,939102
49	0,957489	0,923162	0,951523	0,970475	0,98182
50	0,915279	0,883656	0,858907	0,927378	0,972833
Maximum	0,990371	0,993576	0,992853	0,988167	0,988337
Average	0,83311512	0,771741	0,79950118	0,84245002	0,89523222
Variance	0,01623781	0,02343519	0,01945329	0,01757577	0,01435437

Experiment Optimum 4					
Execution	Results				
	All	One hkd	Two hks	Four hks	No hks
1	0,981039	0,951911	0,994979	0,982255	0,994755
2	0,967192	0,994998	0,993481	0,960952	0,918804
3	0,926273	0,994979	0,953532	0,901394	0,835124
4	0,888598	0,987636	0,952286	0,398565	0,943086
5	0,9136	0,942585	0,99282	0,942039	0,628496
6	0,880689	0,991558	0,652238	0,97871	0,777526
7	0,961624	0,94812	0,910797	0,993819	0,994998
8	0,923595	0,994325	0,992015	0,876	0,820615
9	0,883012	0,84347	0,92992	0,83637	0,918359
10	0,945328	0,992603	0,896382	0,956569	0,935443
11	0,994691	0,99498	0,99496	0,994999	0,993828
12	0,945251	0,975933	0,983648	0,985628	0,797941
13	0,892642	0,878611	0,960852	0,988653	0,553698
14	0,897693	0,988799	0,993729	0,471138	0,876288
15	0,920801	0,994051	0,944515	0,883326	0,833863
16	0,913598	0,849841	0,988205	0,941004	0,883363
17	0,937126	0,91103	0,982689	0,913485	0,945261
18	0,897397	0,976966	0,984615	0,349829	0,981197
19	0,965502	0,991516	0,975184	0,889004	0,99354
20	0,946868	0,991914	0,938315	0,976337	0,88029
21	0,899306	0,924235	0,994815	0,972795	0,283476
22	0,933115	0,927979	0,990442	0,918232	0,884777
23	0,951637	0,903478	0,983541	0,983818	0,923652
24	0,962425	0,993633	0,894767	0,951029	0,993855
25	0,856452	0,994449	0,994598	0,517284	0,616489
26	0,873999	0,978611	0,960863	0,44126	0,916223
27	0,966573	0,994937	0,994999	0,994998	0,866967
28	0,910328	0,976425	0,84226	0,974086	0,826199
29	0,944887	0,955979	0,927468	0,99447	0,885367
30	0,868128	0,959362	0,961742	0,893478	0,541219
31	0,972161	0,993797	0,99322	0,989427	0,913879
32	0,91904	0,990451	0,249756	0,978346	0,895444
33	0,95471	0,972201	0,992859	0,8691	0,994015
34	0,84866	0,994977	0,990981	0,633904	0,481134
35	0,906356	0,86145	0,988849	0,978518	0,696428
36	0,94456	0,976339	0,973099	0,921602	0,890662
37	0,915666	0,933571	0,934412	0,83749	0,956556
38	0,982627	0,95747	0,990936	0,994842	0,987126
39	0,983593	0,99449	0,948334	0,994765	0,994963
40	0,940432	0,980249	0,984019	0,922759	0,85331
41	0,943757	0,994705	0,903508	0,877013	0,993434
42	0,974158	0,924646	0,9913	0,984532	0,993228
43	0,958023	0,939268	0,9159	0,985647	0,991617
44	0,975567	0,944712	0,994895	0,967482	0,994614
45	0,950193	0,968126	0,974344	0,885401	0,959248
46	0,943841	0,96418	0,861274	0,96836	0,966907
47	0,873454	0,955574	0,791441	0,652905	0,954684
48	0,905373	0,899929	0,99498	0,993994	0,596793
49	0,909517	0,978869	0,981925	0,48992	0,970828
50	0,915906	0,959195	0,983041	0,985408	0,597172
Maximum	0,994691	0,994998	0,994999	0,994999	0,994998
Average	0,92933926	0,95978226	0,9399946	0,87545882	0,85853482
Variance	0,00131455	0,00163349	0,01378675	0,03174894	0,02623342

Experiment Optimum 16					
Execution	Results				
	All	One hkd	Two hks	Four hks	No hks
1	0,994451	0,994998	0,994817	0,992841	0,994961
2	0,958457	0,994998	0,99482	0,992598	0,808492
3	0,950539	0,994978	0,993595	0,964452	0,834049
4	0,994164	0,993278	0,994998	0,994133	0,994352
5	0,98983	0,994029	0,994765	0,994517	0,974573
6	0,969417	0,993575	0,994998	0,894902	0,993306
7	0,986783	0,994998	0,991591	0,967319	0,994412
8	0,989266	0,97492	0,994635	0,9913	0,994783
9	0,98919	0,994998	0,994794	0,994707	0,972499
10	0,987784	0,967668	0,994936	0,994998	0,993897
11	0,993125	0,994998	0,994998	0,988833	0,993854
12	0,973215	0,994544	0,994963	0,925842	0,971971
13	0,97556	0,994962	0,994998	0,985022	0,929909
14	0,98966	0,99435	0,994591	0,994998	0,975347
15	0,967896	0,994603	0,994722	0,992811	0,871628
16	0,992243	0,994333	0,993006	0,992734	0,988741
17	0,990178	0,97992	0,993356	0,9938	0,992663
18	0,980561	0,994998	0,993991	0,994611	0,938128
19	0,986624	0,994512	0,994901	0,993738	0,963164
20	0,98887	0,994998	0,994727	0,993998	0,969899
21	0,993139	0,994998	0,988455	0,994321	0,994603
22	0,99361	0,994787	0,994779	0,992208	0,992842
23	0,993807	0,993947	0,993509	0,993451	0,994242
24	0,99391	0,99454	0,994998	0,991937	0,994321
25	0,981067	0,994998	0,994998	0,943503	0,988293
26	0,994484	0,994998	0,994085	0,993799	0,994962
27	0,990694	0,994998	0,994704	0,993285	0,979698
28	0,994709	0,994608	0,99425	0,994998	0,994945
29	0,99179	0,985489	0,994074	0,99458	0,992983
30	0,965471	0,993493	0,994628	0,994698	0,851391
31	0,985461	0,994998	0,994578	0,994998	0,956202
32	0,991829	0,994662	0,994537	0,994998	0,982959
33	0,989074	0,988398	0,984923	0,991904	0,990545
34	0,977361	0,994695	0,994117	0,930739	0,994553
35	0,991382	0,994678	0,994089	0,99487	0,981011
36	0,992072	0,99498	0,983615	0,994923	0,994389
37	0,983654	0,994998	0,994998	0,994835	0,947973
38	0,993274	0,994945	0,994445	0,994422	0,989331
39	0,981522	0,994998	0,994434	0,994609	0,940986
40	0,99478	0,994807	0,994578	0,994801	0,994943
41	0,994737	0,994669	0,994724	0,994538	0,994998
42	0,986755	0,9859	0,993506	0,973492	0,994021
43	0,98804	0,994962	0,970151	0,992774	0,994298
44	0,969084	0,994978	0,994498	0,882512	0,99441
45	0,994921	0,994998	0,994998	0,994697	0,994998
46	0,994686	0,994998	0,994678	0,994863	0,994245
47	0,991508	0,98514	0,994136	0,993715	0,993709
48	0,983179	0,945458	0,994822	0,993836	0,99452
49	0,994653	0,994794	0,994771	0,994789	0,99427
50	0,971311	0,993357	0,980327	0,991363	0,916503
Maximum	0,994921	0,994998	0,994998	0,994998	0,994998
Average	0,98579554	0,99177854	0,99315214	0,98425224	0,97135544
Variance	0,00010812	7,4319E-05	1,9776E-05	0,00062973	0,0018994

Experiment Optimum 64					
Execution	Results				
	All	One hkd	Two hks	Four hks	No hks
1	0,994964	0,994998	0,994998	0,994998	0,994869
2	0,994701	0,994998	0,994998	0,994998	0,99388
3	0,994745	0,994998	0,994998	0,994998	0,993994
4	0,994757	0,994998	0,994997	0,994998	0,993911
5	0,994379	0,994998	0,994982	0,993683	0,993847
6	0,994689	0,994998	0,994998	0,993853	0,994934
7	0,994928	0,994998	0,994998	0,994998	0,994718
8	0,994335	0,994998	0,994998	0,993177	0,994291
9	0,994164	0,994998	0,994806	0,993918	0,992829
10	0,994876	0,994998	0,994998	0,994899	0,994619
11	0,994614	0,994998	0,994897	0,994061	0,994529
12	0,993702	0,994998	0,994998	0,991738	0,993091
13	0,994793	0,994967	0,994998	0,994606	0,994596
14	0,994693	0,994998	0,994998	0,994678	0,994106
15	0,994749	0,994998	0,994998	0,994349	0,994652
16	0,994789	0,994998	0,994982	0,994493	0,994692
17	0,994578	0,994998	0,994998	0,993795	0,994522
18	0,994948	0,994998	0,994998	0,994835	0,994982
19	0,99488	0,994998	0,994998	0,994831	0,994711
20	0,9949	0,994998	0,994998	0,994706	0,994921
21	0,994741	0,994998	0,994998	0,994998	0,993979
22	0,994347	0,994998	0,994998	0,993695	0,993679
23	0,994865	0,994998	0,994998	0,994795	0,994681
24	0,994506	0,994998	0,994998	0,993486	0,994394
25	0,994482	0,994998	0,994935	0,994303	0,993646
26	0,994709	0,994997	0,994998	0,994871	0,993945
27	0,994208	0,994998	0,994998	0,994998	0,991572
28	0,994673	0,994998	0,994997	0,994709	0,993972
29	0,994813	0,994998	0,994998	0,994695	0,994576
30	0,994642	0,994998	0,994998	0,994041	0,99448
31	0,994992	0,994998	0,994998	0,994982	0,994998
32	0,994681	0,994998	0,994998	0,994821	0,993887
33	0,994542	0,994998	0,994998	0,994596	0,99374
34	0,994944	0,994998	0,994998	0,994831	0,994953
35	0,994825	0,994998	0,994998	0,994998	0,994313
36	0,994797	0,994998	0,994998	0,994462	0,994751
37	0,994697	0,994998	0,994966	0,994086	0,99472
38	0,994912	0,994998	0,994998	0,994905	0,994747
39	0,994386	0,994998	0,994998	0,994705	0,992862
40	0,994586	0,994998	0,994998	0,994898	0,993491
41	0,994757	0,994998	0,994998	0,994998	0,99406
42	0,994486	0,994998	0,994998	0,993704	0,99429
43	0,994638	0,994998	0,994857	0,993997	0,994749
44	0,99457	0,994998	0,994998	0,993988	0,994325
45	0,988251	0,994997	0,994998	0,994949	0,965018
46	0,994112	0,994998	0,994998	0,993088	0,993259
47	0,990309	0,994998	0,994998	0,994829	0,977759
48	0,994586	0,994998	0,994753	0,994614	0,993991
49	0,994765	0,994998	0,994923	0,994953	0,994245
50	0,994669	0,994998	0,994344	0,994921	0,994418
Maximum	0,994992	0,994998	0,994998	0,994998	0,994998
Average	0,9944335	0,99499734	0,99496726	0,99445056	0,99328388
Variance	1,2338E-06	1,9209E-11	1,0583E-08	4,3192E-07	2,2474E-05