Department of Economics "S. Cognetti de Martiis"

Via Po, 53 – 10124 Torino (Italy)
Tel. (+39) 011 6704917 - Fax (+39) 011 6703895
URL: http//www.de.unito.it

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UNVEILING DIFFUSION DYNAMICS: AN AUTOCATALYTIC PERCOLATION MODEL OF ENVIRONMENTAL INNOVATION DIFFUSION AND THE OPTIMAL DYNAMIC PATH OF ADOPTION SUBSIDIES

Simona Cantono

Dipartimento di Economia "S. Cognetti de Martiis"

LEI & BRICK - Laboratorio di economia dell'innovazione "Franco Momigliano"
Bureau of Research in Innovation, Complexity and Knowledge, Collegio Carlo Alberto

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Unveiling diffusion dynamics: an autocatalytic percolation model of environmental innovation diffusion and the optimal dynamic path of adoption subsidies

Simona Cantono*

Abstract

This article applies the autocatalytic percolation model developed by Cantono and Solomon (2010) to the diffusion of environmental innovation. It contributes to the recent applied microeconomic diffusion literature by unveiling diffusion dynamics, by determining under what conditions is diffusion self-sustaining and by defining the optimal dynamic schedule of adoption subsidies which insures autonomous propagation. To this end a model which combines in a unique framework a learning curve model of dynamic cost reductions, a discrete choice model of heterogeneous technology adoption and a contagion model of technology diffusion is developed. It is shown that the system dynamics are discontinuous, path-dependent and irreversible. Propagation dynamics are uncovered: diffusion occurs along subsequent conquers of islands of potential adopters. Under certain circumstances diffusion is self-sustaining. In other occasions diffusion is confined to a negligible sub-set of the entire population of potential adopters. In the latter case a policy intervention can drive the system to overall propagation. This can be achieved by adoption subsidies which, in order to be effective and to avoid a waste of resources, must follow an optimal dynamic schedule. It is shown that the phasing-out stage is as important as the early stage of the intervention.

JEL Classification: C63, H23, O33

Keywords: innovation diffusion, adoption subsidies, percolation, heterogeneous agents

* Department of Economics "S. Cognetti de Martiis", University of Torino. E-mail: simona.cantono@unito.it

I. INTRODUCTION

Time-delayed diffusion paths have been challenging economists since Griliches (1957) seminal contribution. Innovations spread initially slowly because of a combinations of factors ranging, broadly speaking, from epidemic-like contagion mechanisms, to information problems, heterogeneity, uncertainty and expectations (Mansfield, 1961, 1968; Bass, 1969; David, 1969; Davies, 1979; Stoneman, 1980). Although researchers have been acquainted with the notion of barriers to overall propagation, nonetheless the phase of diffusion still arouses interest today especially when dealing with environmentally friendly technologies. A considerable collection of scientific articles has investigated the reasons lying behind the discrete extent of success of environmental innovations[1]. Some obstacles have been detected and several policy actions have been directed to remove them. Yet the territory conquered so far by environmentally friendly technologies looks limited if compared to the relevant role played by their widespread adoption in the delicate relationship between economic growth and environmental sustainability (Jaffe et. Al. 2003; Jaffe et. Al., 2005; Popp et al., 2009). In addition to that, their policy domain shows signs of hysteria which manifest themselves either through firms' fear of a solitary future not anymore sustained by public interventions or through politicians' fear of failure who see their taxpayers' money wasted on presumed unsuccessful trials.

Drawing on the applied microeconomic diffusion literature this paper deals with the diffusion of environmental technologies and its policy implications.

The fact that many environmental beneficial technologies do not spread in the market despite their cost-effectiveness has been analyzed by Jaffe and Stavins (1995) in one of their most celebrated article. Through a rational choice model of technology adoption, they showed how the meager result achieved by energy-saving technologies may be due to high-up front adoption costs and to principal / agent problems. Heterogeneity is in fact unlikely to explain alone time-delayed propagation paths. High up-front costs and information related issues inflict additional agony. Their model has been successfully extended to the analysis of learning curve cost reductions (Isoard and Soria, 2001; Soderholm and Klaassen, 2007). Eco-innovations diffusion may be enhanced by future cost reductions which, if free to work, could mitigate the considerable burden imposed by high initial costs. Yet learning curves seem insufficient, even despite favourable and promising estimations (McDonald and Schrattenholzer, 2000). One possible candidate for an explanation is the limited extent of information transfers from past to future adopters. Barriers to the flow of information could delay the initial phase of the propagation process thus limiting the effect of learning curves by confining the wave of diffusion to a negligible subset of the population of potential adopters. As it will be shown in this paper this might indeed be the case.

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[1] See Kemp (1997), Carraro (1999), Van Den Bergh and Gowdy (2000), Janssen and Jager (2002), Kobos et. Al. (2006), Pan and Kohler (2007), Faber and Frenken (2009), Sterner and Turnheim (2009), Faber et. Al. (2010), Soderholm and Pettersson (2011), Woersdorfer and Kaus (2011). The list is not pretended to be exhaustive. For further references see also Rennings (2000) and Jaffe et. Al. (2003). For a comprehensive view of technological diffusion see Stoneman (2002).

Generally defined as peer-to-peer mechanism, the relevance of understanding interdependence among economic agents in terms of local transfers of information / knowledge not globally available makes it a topical subject matter, and it is often modelled through the aid of social / economic networks games (Cowan and Jonard, 2004; Morris, 2000). Here a different approach is taken. By drawing on several successful marriages between percolation theory and the analysis of the propagation of new technologies (Antonelli, 1996; Silverberg and Verspagen, 2005, Frenken et. Al. 2008; Hohnisch et. Al., 2008; Cantono and Silverberg, 2009; Frenken et. Al., 2009) the model presented in this paper applies the autocatalytic percolation framework developed by Cantono and Solomon (2010) to the diffusion of environmental innovations and it investigates its policy implications.

The first contribution of the present work is to uncover diffusion dynamics. To this end a model which combines in a unique framework a learning curve model of dynamic cost reductions, a discrete choice model of heterogeneous technology adoption and a contagion model of technology diffusion is developed. In such an environment, the underlying dynamics of the propagation process are unveiled: diffusion occurs along narrow channels and sudden explosions, i.e. through a series of isolated communities of potential adopters. Highlighted in the past (Karshenas and Stoneman, 1993) and recently contextualized in the domain of demand-induced innovations (Foellmi and Zweimuller, 2006), non-mixing populations emerge in the present model out of the intertwining between potential adopters' heterogeneity and the network of interactions through which information flows. Formally, it is assumed that there exist a new technology ready to be commercialized and available to a heterogeneous population of firms located in a business network. The new technology is a new cleaner production process embedded in a capital good supplied by a monopolist (for example an energy saving technology). The adoption cost is common knowledge and decreases according to learning curve costs reductions. Diffusion is endogenous, there is a positive feedback loop between the cumulative number of adopters and the pace of learning economies: diffusion depends on costs reductions which in turn depends on the extent of diffusion[2]. Firms receive information about the technology by contact with past adopters which are their business partners and compare their private information about the benefit from adoption, which are heterogeneous, with the cost. However only when the former is greater than the latter will a firm adopt the new technology. In other words adoption takes place not only by information transmission but it is also the outcome of a rational, though basic, decision process. While in this paper the most simple mechanism of processing information is assumed (i.e. direct transfer) the growing field of social learning is endowed with a set of significant models able to explain how the way in which information is processed may influence the choice of adoption (Bandiera and Rasul, 2006; Krishnan

[2] While it is true that in this sense diffusion is endogenous, costs reductions depend on the extent of diffusion at a constant rate (i.e. the learning coefficient) which is a parameter of the model. In other words we do not model a simultaneous innovation – diffusion model that would require an explanation on how R&D efforts impact the pace of diffusion through the extent of learning economies. Yet it is an important issue as shown by Soderholm and Klaassen (2007).

and Shubba, 2009; Conley and Udry, 2010). How sensitive the results presented in this paper are to the inclusion of social learning is a challenging question left to future work.

A second contribution of this paper is that of defining the emergent aggregate equilibria of the system thus, implicitly determining under what conditions is diffusion self-sustaining. It will be shown how the intertwining between learning curve costs reductions, heterogeneity and the network of interactions through which information flows can generate a powerful autocatalytic feedback loop: given an arbitrary initial level of the adoption cost for a given initial number of adopters, a certain fraction of individuals become adopters; this in turn will result in a decrease in the adoption cost which will lead to an increase in the density of potential adopters, which in turn will cause an increase in the number of adopters, which will trigger a further decrease in the adoption cost, and so on. The autocatalytic feedback loop between the density of potential adopters, the increase in the extent of diffusion and endogenous cost reductions renders the system dynamics discontinuous, path-dependent and irreversible. Under certain conditions the regions of the system dynamics are characterized either by a negligible level of propagation which hangs up at an apparently stable state ready to be spurred by a minimum disturbance, or by an insignificant conquest because entrapped in isolated communities of adopters, or by overall propagation. Once the critical mass is reached the pace of learning economies along with diffusion clusters' size dynamics enhance a self-sustaining propagation process.

Diffusion dynamics are rarely debated by the scientific community thanks to the general consent around the logistic function. However there is a renewed interest on the topic as documented by important contributions related to our work. In the spirit of the pioneering work by Galeotti (2006), Jackson and Yariv (2007) investigate the dynamics of the diffusion of strategic behavior and they characterize the equilibrium properties of the system in a network game. By assuming different mechanisms and by employing a diverse methodology it is remarkable to note that we come to very similar conclusions[3]. Young (2009) extends the classes of epidemic, social influence and social learning models to heterogeneous populations of potential adopters. He founds that the characteristic dynamics exhibited by the three classes of models survive to the introduction of heterogeneity and that they may be employed alternatively depending on the specific application. While the contribution offered by the present paper is somehow more specific, it does however improve on one of the question left unanswered and highlighted by Young himself, i.e. the influence of the introduction of a network structure. His epidemic-type of model (the closest to that proposed here) cannot explain the case in which the ideal equilibrium is not reached. New technologies that would otherwise conquer a large set of adopters are often limited to a much smaller set even if the set of potential adopters is very large. This might be due, as the present work illustrates, to limited communication within the system which impede to the diffusion process to reach some (or many) of the potential adopters.

[3] Showing why it is so it is beyond the scope of the present work. However, at a first sight, the interaction between heterogeneity and the network structure through which information flows (whether modelled through network games or through percolation) seems to mark an undeletable sign on the properties of aggregate equilibria.

Entrapped by isolated clusters of adopters and timidly incited by favourable learning economies, new technologies may end up in a never-ending struggle for freedom. In that case, only an external intervention could spur the propagation process or the new technology would be left to its destiny. Fair enough, if it concerns diffusion in a perfect competitive market where adopters take their decisions aided by complete knowledge and perfect foresight. Problematic otherwise in an a priori biased environment. That of planning and scheduling an optimal strategy for policy support towards environmentally friendly technologies is indeed a cumbersome issue (Requate and Unold, 2003; Snyder et. Al., 2003; Kemp and Pontoglio, 2011; Timilsina et. Al., 2011). The present work is limited to the analysis of adoption subsidies. Being an element of the set of market-based instruments, adoption subsidies benefit from the merits of their class (Jaffe et al., 2005). In the face of high up-front costs they look dynamically effective (Jaffe and Stavins, 1995). Although it might be difficult to chose the best available technological options even if subsidies were the correct policy to implement (Soderholm and Pettersson, 2011), nonetheless their wide real applications need scientific inquiries to sustain decision makers and the need is felt the most in the phasing-out stage. The third contribution of this work is thus that of defining the optimal dynamic schedule of subsidies which insures self-sustaining diffusion. In the regions of the system dynamics characterized by microscopic diffusion a dynamic policy of such a type could accompany the process until the moment it becomes self-sustaining and the model offered here depicts the optimal path which if over-reproduced would lead to an excessive use of resources and, at the same time, it would not guarantee diffusion for not only the extent of the policy but also its length matters; if under-reproduced would lead to a failure and the process would simply stop.

The remainder of the paper is organized as follows. Section II describes in details the application of the autocatalytic percolation model to the diffusion of environmental technologies. Section III illustrates the main dynamics of the model. Section IV presents the optimal dynamic schedule of adoption subsidies. Section V illustrates the results and implications from the Monte Carlo simulations. Section VI offers the conclusions and future outlook.

II. THE MATHEMATICAL FORMULATION OF THE MODEL AND ITS PRELIMINARY RESULTS

By drawing on the model developed by Cantono and Solomon (2010), this section illustrates the mathematical formulation of the autocatalytic percolation model of environmental technology diffusion.

It has been argued in the introduction that one of the reason lying behind time-delayed diffusion path of environmental technologies may be the existence of barriers to the free flow of information. Although the presence of an endogenous learning mechanism in the decision process is not peculiar to the adoption of environmental technologies, nonetheless it is realistic to assume that at least in certain relevant cases there are internal feedbacks between past and future adopters concerning the performance and reliability of such a kind of technologies and the more they are free to flow, the more efficient and cost-effective opportunities may be selected. Instead of focusing on the way in which information is processed (and how it consequently influences the choice of adoption), here the attention is turned to the pattern of information transmission in order to understand how it interacts with the other mechanisms reproduced by the model (i.e. heterogeneity and endogenous cost reductions) and, ultimately impacts on diffusion dynamics. To this end a simple mechanism of information processing is assumed, that of direct transfer through contact between neighbours. Consider a set of firms labelled by an index $i = 1, 2, ..., M$. Firms are located at each node of a business network and are connected through the links i, j which form the network geometry. A bi-dimensional regular lattice (Ising network) is assumed as the network structure. Information about the new cleaner production process spills over the business network through contact between business partners. The first necessary, but not sufficient, condition for adoption is thus defined:

1. a firm waits until it receives a signal from at least a business partner who has already adopted.

The diffusion of innovation is not a wave which overwhelms unconscious potential adopters. Once available, it does not spread in the market frictionless, as if it were a pure epidemic disease. It rather encounters a collection of connected heterogeneous individuals who perceive the advantages from adoption differently. In other words the extent of diffusion depends on the outcome of the adoption decision which is taken by each firm out of the comparison between their heterogeneous advantages and the disadvantages from adoption. In the present model firms heterogeneity is characterized by their gross benefit from adoption b. The $b_i s$ are random independent variables drawn by a Pareto power law probability distribution:

$$\mathrm{P}(b_i > \theta) = \theta^{-\mu} \text{ for } \theta > 1 \text{ and } \mathrm{P}(b_i > \theta) = 1 \text{ for } \theta < 1$$

The choice of the probability distribution is based on the assumption that there is a positive relation between firms' performance and their willingness to adopt new technologies. Firms' performance has been found to display such power laws distribution (Axtell, 2001)[4]. Firms with a relatively high willingness to adopt will approach the time of adoption earlier. Their structures of production may perform better in catching-up with innovations. The benefits may exceed the costs of renouncing to expected future cost reductions. On the contrary, firms characterized by a relatively low gross benefit are those firms who delay the adoption process.

The decision rule, which is also the second condition for adoption, is thus defined as follows:

2. firm i adopts if and only if $b_i > c_k$ (where $c_k > 0$ is the cost of adoption at time k, known to the population as a whole)[5].

In other words, adoption is a best response strategy if $b_i > c_k$ conditional on receiving a signal from at least a business partner who has already adopted[6]. The status of potential adopter is also defined:

[4] Substituting Eq. 1 with any exponential would not change the qualitative behaviour of the system.

[5] The influence of important factors on the individual profitability of adoption, such as factor prices (Acemoglu and Filknstein, 2008), cannot be captured. Though in the case of energy-saving technologies, for example, factor prices seem to influence more the direction rather than the pace of diffusion (Jaffe and Stavins, 1995; Newel et al, 1999; Linn, 2008).

a firm i is a potential adopter iff $b_i > c_k$. Following the assumptions above, the density of potential adopters at any given level of c_k is:

$$\rho_k = c_k^{-\mu} \text{ for } c_k > 1 \quad [1]$$

Eq. 1 reproduces the first important mechanism of the model which expresses the influence of the system, represented by the global variable c_k, on the status of its components, reflected by ρ_k. According to percolation theory there exists a critical density ρ_c such that for any $\rho_k > \rho_c$, percolation takes place with probability one, i.e. the new technology conquers the entire market ($\rho_c = 0.592$ in a bi-dimensional lattice; see Stauffer and Aharony, 1994). At $\rho_k = \rho_c$, $c_k \equiv c_c \Rightarrow$ $c_c = \rho_c^{-1/\mu}$ which implicitly defines the percolation critical cost c_c. The usual percolation setting would thus suggest that for any initial level of the cost c_0 which is lower than the percolation critical threshold c_c diffusion takes off unconditionally. Moreover the phase transition at $c_0 = c_c$ is continuous. However by introducing the mechanism of learning curve cost reductions diffusion dynamics changes in nature. As it will be shown in the next section, the phase transition at the new critical value $c_{0,c}$ is discontinuous.

Let us introduce the bottom-up mechanism of the model according to which the global status of the system described by c_k changes because of the changes in the status of its component, N_k (i.e. because of the number of firms that have switched from potential adopters to adopters). Given its initial level c_0, the cost of adoption is expected to decrease exponentially with the increase in the cumulative number of adopters N_k according to:

$$c_k = c_0 N_{k-1}^{-\alpha} \quad [2]$$

where $0 \le \alpha \le 1$ is the learning coefficient[7].

Only one mechanism is missing, the peer-to-peer mechanism which expresses the influence of the network of interactions on diffusion dynamics. Due the intertwining between heterogeneity and the network structure, not all the susceptible firms become adopters but only a certain fraction according to:

$$N_k = N_0 \left(1 - \frac{\rho_k}{\rho_c}\right)^{-\gamma} \quad [3]$$

Eq. 3 defines the dynamics of the propagation process in terms of the cumulative number of adopters N_k[8] (i.e. the number of *infected* firms) at each iteration k, as a function of the initial

[6] From this point of view it is clear how simplifying the hypothesis of direct transfer through contact is. Social learning models relax this assumption and offer, among other things, the analysis of the mechanisms underlying information processing which could be included in the present framework and lead to interesting results and implications.

[7] In Equation 2 the learning curve cost reduction is represented by its classic functional form. However recent studies have proven that it may not always be the appropriate one (Pan and Kohler, 2007).

[8] We measure diffusion by the extensive margin of adoption: each decision to adopt corresponds to one more adopter and it adds up to the cumulative number of adopters. However this implies that the environmental impact of eco-innovations diffusion cannot be analyzed. Think for instance, like in the present case, to the spread of a new cleaner production process. The extent of pollution reduction does not only depend on whether a firm has adopted the

exogenous number of adopters N_0 ($N_0 > 0$), the susceptibility coefficient γ (which expresses the architecture of the network) and the density of potential adopters ρ (i.e. the number of *susceptible* firms in the business network). The mean size of the percolation cluster (i.e. the collection of adopters) diverges when the density of potential adopters ρ_k approaches the critical density ρ_c. At $\rho_k = 0$, $N_k = N_0$. Through the critical density ρ_c and the susceptibility critical exponent $\gamma \geq 0$, Eq. 3 shows how the size of the diffusion cluster depends on the network of interactions[9].

The intertwining between heterogeneity, endogenous costs reductions and the network of interactions through which information flows give raise to an autocatalytic feedback loop. Given the initial cost of adoption c_0 and the initial exogenous number of adopters N_0, the cost c decreases according to Eq. 2. An increase in the density of potential adopters ρ follows (Eq. 1), which in turn increases the number of cumulative adopters N (Eq. 3). The further decrease of the cost of adoption leads to a further increase in the density of potential adopters and, as a consequence, to a further raise in the cumulative number of adopters and so on. Formally, for $k > 0$, the autocatalytic feedback loop can be written as:

$$c_1 = c_0 N_0^{-\alpha} \rightarrow \rho_1 = c_1^{-\mu} \rightarrow N_1 = N_0 [1 - \rho_1 / \rho_c]^{-\gamma} \rightarrow c_2 = c_0 N_1^{-\alpha} \rightarrow \rho_2 = c_2^{-\mu} \rightarrow ....$$

The iterative process starts at $k = 1$. At $k = 0$, $c_k = c_0$, $\rho_k = 0$ and $N_k = N_0$.

For certain values of the parameters c_0, α, γ and μ the process initiated by the N_0 seeds may fall in very diverse regions of the system dynamics. However, due to the symmetries of the problem,

$$N(N_0, c_0, \alpha, \mu) = N\left(N_0, c_0' = c_0^{\mu/\mu'}, \alpha' = \frac{\alpha^{\mu}}{\mu'}, \mu' \right)$$

$$N(N_0, c_0, \alpha, \mu) = \frac{N_0}{N_0'} N\left(N_0', c_0' = \left(\frac{N_0'}{N_0} \right)^{\alpha} c_0, \alpha, \mu \right)$$

only two of the four parameters N_0, c_0, α and μ are independent (while the critical exponent $\gamma = 2.11$ as shown by Cantono and Solomon, 2010): the effects of varying μ and N_0 can be related to the effects of varying c_0 and α (in the remainder of the paper μ and N_0 are maintained fixed - $\mu = 5/2; N_0 = 40$ - and mainly c_0 and α will vary).

Finally, the problem: given μ, N_0, for which combination of the initial cost of adoption c_0 and the learning coefficient α will the new technology conquer the entire lattice. In particular, which combinations $c_{0,c}, \alpha$ separate between regimes of macroscopic and microscopic diffusion?

The results show that:

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technology, but also on how intensively that technology is used by the same firm, i.e. the share of pollution reduction per unit of output multiplied by cumulative output. The former is the extensive margin of adoption while the latter is the intensive margin of adoption. Any further research in this direction is likely to produce useful results

[9] The goodness of fit of Eq. 3, which becomes an approximation of the clusters size dynamics when $N_0 > 1$, has been checked by confronting the theoretical predictions with the Monte Carlo simulations and the results of the comparison are reassuring: the scaling hypothesis is guaranteed (Cantono and Solomon, 2010).

- for values $c_o < c_{0,c}(\mu, N_0, \alpha, \gamma)$ the new technology propagates frictionless until overall diffusion is reached;
- for values $c_o \sim c_{0,c}(\mu, N_0, \alpha, \gamma)$ the process hangs up at a finite N, ready to take-off at the first minimum external boost;
- for values $c_o > c_{0,c}(\mu, N_0, \alpha, \gamma)$, i.e. high up-front costs, the process rests dormant in a stagnation region, attracted towards a lower bound propagation level, unless exogenously driven until an upper bound propagation level after which it would diverge to overall diffusion.

III. THE AUTOCATALYTIC FEEDBACK LOOP AND ITS MAIN DYNAMICS

Let us investigate formally the dynamics of the model. We were left with the iterative dynamic process of the form:

$$c_k = c_0 N_{k-1}^{-\alpha} \qquad \rho_k = c_k^{-\mu}$$

$$N_k = N_0 \left(1 - \frac{\rho_k}{\rho_c}\right)^{-\gamma}$$

Figure 1: the autocatalytic feedback loop

By substituting c_k (Eq. 2) into ρ_k (Eq. 1) and further substituting into N_k (Eq. 3) the following equation can be obtained:

$$N_k = N_0 \left[1 - \left(\frac{c_c}{c_0}\right)^{\mu} N_{k-1}^{\alpha\mu}\right]^{-\gamma} \qquad [4]$$

The outcome of the iterative process (Eq. 1, 2, 3) is found by looking for the fixed points which fulfil the stationary condition $N_k = N_{k-1}$ in Eq. 4:

$$N = N_0 \left[1 - \left(\frac{c_c}{c_0}\right)^{\mu} N^{\alpha\mu}\right]^{-\gamma} \qquad [5]$$

A graphical solution reveals its main dynamics (Figure 2). The autocatalytic feedback loop triggered by the iterative process affects the percolation phase transition rendering the system dynamics discontinuous, path-dependent and irreversible. Given the set of parameters μ, N_0, γ and α, depending on the initial cost of adoption c_0 the process may fall in very different states. There exists a critical value of the initial cost $c_{0,c}$ for which Eq. 5 has a unique tangent solution (Fig. 2/a), the repulsive fixed point $N_{hang-up}$. High up-front costs of adoption seem enough to dominate learning curve cost reductions and the process hangs up at a negligible level of diffusion but the stability of the system is an illusion for the slightest disturbance could trigger the process to overall propagation. For any $c_0 < c_{0,c}$ the new technology conquers the entire system, Eq. 5 does not have real solutions (Fig. 2/b). If $c_0 > c_{0,c}$ then Eq. 5 has two solutions (Fig. 2/c): the attractive fixed point N_{stop} and the repulsive fixed point $N_{no\ return}$. The process initiated by the N_0 seeds is destined to stop very soon at N_{stop}. Only if pushed beyond $N_{no\ return}$ will diffusion be self-sustaining.

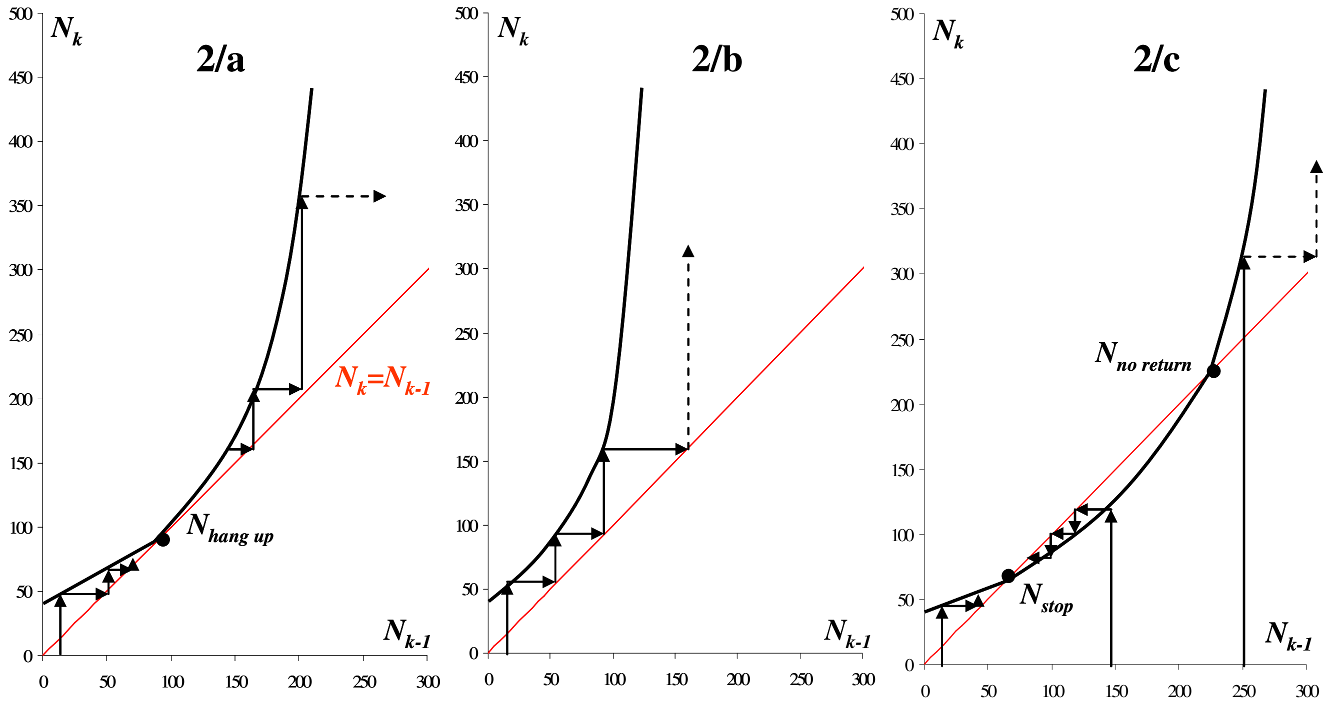

Figure 2: Graphical solution of Eq. 4 for $\mu = 2/5$, $N_0 = 40$, $\gamma = 2.11$, $\alpha = 0.3$ and $c_c = 1.23$; $c_0 \sim c_{0,c}$ in 1/a, $c_0 < c_{0,c}$ in 1/b and $c_0 > c_{0,c}$ in 1/c

By looking for the critical values of c_0, i.e. $c_{0,c}$ which separates between negligible and macroscopic diffusion the first order phase transition in the c_0, α plane emerges. As Figure 3 describes, all the combinations c_0, α falling in the area subtended by the curve are those for which Eq. 5 does not have real solutions. The points on the curve represent all the situations where there is the tangent fixed point solution. Microscopic propagation is the result for any combination of c_0, α falling above the curve. The latter case is the case in which Eq. 5 displays two fixed points solutions.

In the absence of learning economies (i.e. $\alpha = 0$ in Figure 3) diffusion takes off only if $c_0 < c_c$, as it would be the case in the traditional physics inspired percolation settings. As opposed to the usual percolation phase transition $c_0 = c_c$, the phase transition at the new critical value $c_{0,c}$ is discontinuous, and $c_{0,c} > c_c$. In the presence of endogenous cost reductions the autocatalytic feedback look may drive the process to overall propagation even if $c_0 \geq c_c$, it thus impacts on diffusion dynamics by shifting the percolation critical threshold forward. As argued by the literature, learning curve cost reductions mitigate the impact of high up-front costs and may help the new technology to overcome the early obstacles to its propagation (Isoard and Soria, 2001;

Soderholm and Klaassen, 2007). However there are cases in which endogenous costs reductions are insufficient.

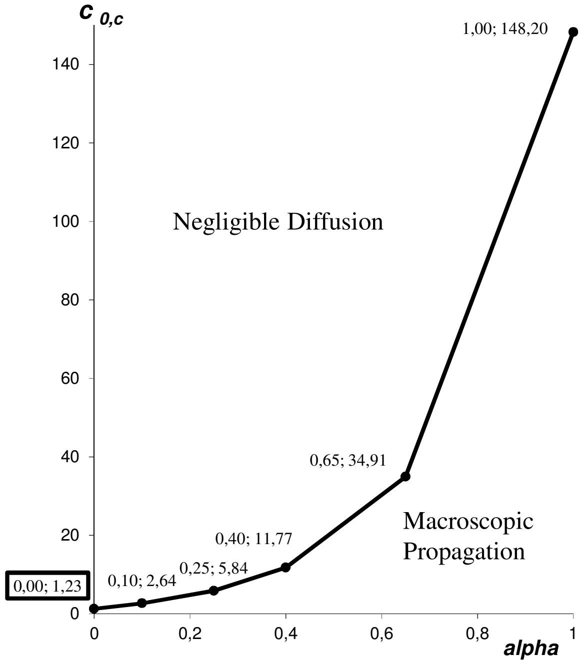

Figure 3: phase transition in the c_0, α **plane** ($\mu = 2/5$, $N_0 = 40$, $\gamma = 2.11$)

Let us illustrate the regions of the system dynamics as a function of N and c_0 (Fig. 4). The particular combination of α, μ and γ which has been chosen allows for a simple analytical solution of Eq. 5 (for $\alpha \mu \gamma = 1$, $c_{0,c} = 4^{1/\mu} c_c N_0^{\alpha}$ and $N_{hang-up} = 2^{\gamma} N_0$ as shown by the Appendix). Figure 4 shows that for values of $c_0 < c_{0,c}$ the system falls under the regime of macroscopic propagation. The process initiated by the N_0 seeds conquers undisturbed the entire market, i.e. diffusion is self-sustaining. At $c_0 = c_{0,c} = 4^{1/\mu} c_c N_0^{\alpha}$ suddenly a real solution appears and the process hangs up at $N_{hang-up} = 2^{\gamma} N_0$. For every value of $c_0 > c_{0,c}$ the process boils down to negligible diffusion attracted by the lower bound solution frontier N_{stop}. Because of the interaction between high up front costs, potential adopters heterogeneity and the network structure not all the firms potentially willing to adopt are reached by the diffusion wave instantaneously but only an insignificant fraction. Entrapped by isolated clusters of adopters, the diffusion process cannot benefit from learning curve cost reductions. There is no way of reaching self-sustained diffusion other than that of crossing the stagnation region supported by external intervention. The next section shows how.

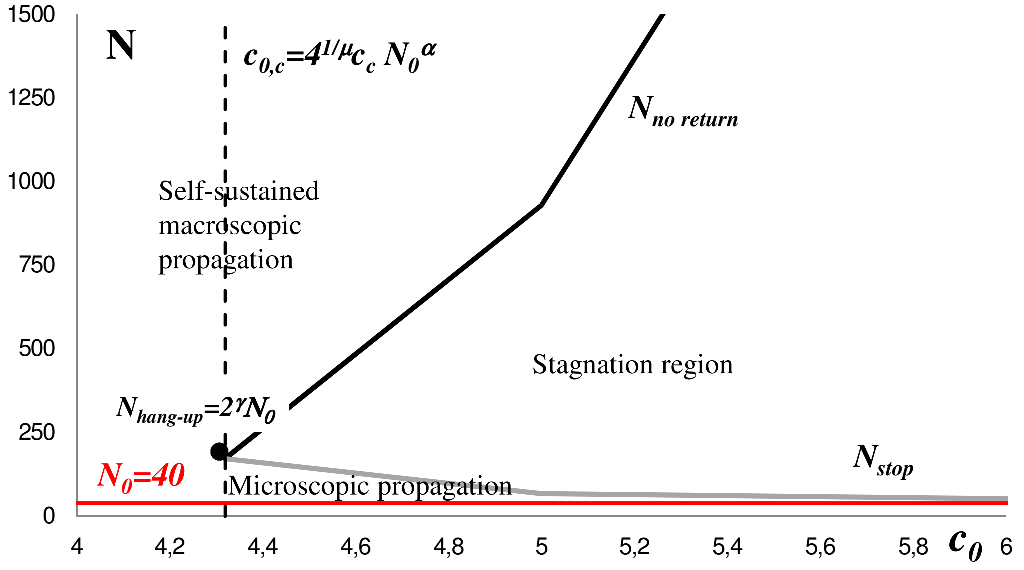

Figure 4: the regions of the system dynamics as a function of N **and** c_0**;** $N_0 = 40$**,** $\alpha = 0.19, \gamma = 2.11$ **and** $\mu = 2.5$

IV. THE OPTIMAL DYNAMIC PATH OF SUBSIDY

There are several instruments which, in the model environment, would allow a new technology entrapped by isolated communities of adopters to diffuse. One possible policy response could be that of acting either on the learning coefficient or on the initial cost of adoption. That may be accomplished by policy interventions in favour of R&D. Another possibility is that of favouring the spread of information, for example by totally sustaining $N_0 >> 0$ initial adopters. The present work is limited to the case of adoption subsidies and it aims to compute the optimal dynamic schedule which guarantees self-sustained propagation. The analysis is not extended to welfare implications which is left to future work. Indeed it is known that a subsidy policy in a monopoly regime may not always yield welfare gains (Stoneman and David, 1986). Here it is just assumed that the policy passes that test.

Many patterns of subsidies are possible, but only one can be implemented with the minimum effort. Let us define the optimal (necessary and sufficient) dynamic path of subsidies S in the case of high-up front cost, i.e. $c_o > c_{0,c}(\mu, N_0, \alpha)$.

In order to be of size N_k, the percolation cluster requires a level of the cost:

$$c_k = c_c\left[1 - \left(N_k / N_0\right)^{-1/\gamma}\right]^{-1/\mu} \qquad [6]$$

Eq. 6 is obtained by substituting ρ_k (Eq. 1) into N_k (Eq. 3) and further solving for c_k.

The most effective and minimal intervention S_k which fills the gap between the cost necessary to reach N_k adopters, given by Eq. 6, and the cost insured by N_{k-1} adopters, provided by Eq. 2, is:

$$S_k = c_0 N_{k-1}^{-\alpha} - c_c \left[1 - \left(\frac{N_k}{N_0} \right)^{-1/\gamma} \right]^{-1/\mu} \qquad [7]$$

Figure 5 illustrates c as a function of N according to both Eq. 2 and Eq. 6. If there are not points of intersection between Eq. 2 and Eq. 6 the system is back to the situation of macroscopic diffusion: $c_0 < c_{0,c}(\mu, N_0, \alpha)$ and public intervention is not necessary ($S_k < 0$ for each k): the process initiated by the N_0 seeds never stops. The tangent solution illustrates the case in which propagation hangs up at a small finite $N = N_{hang-up}$, and $c_o \sim c_{0,c}(\mu, N_0, \alpha)$. Any positive level of subsidies given even only to one potential adopter would be enough to overcome the single repulsive fixed point solution. Finally if the two curves intersect each other twice, $c_o > c_{0,c}(\mu, N_0, \alpha)$, then subsidies are needed in the interval between the intersection points, i.e. between N_{stop} and $N_{no\ return}$, or diffusion will never take-off.

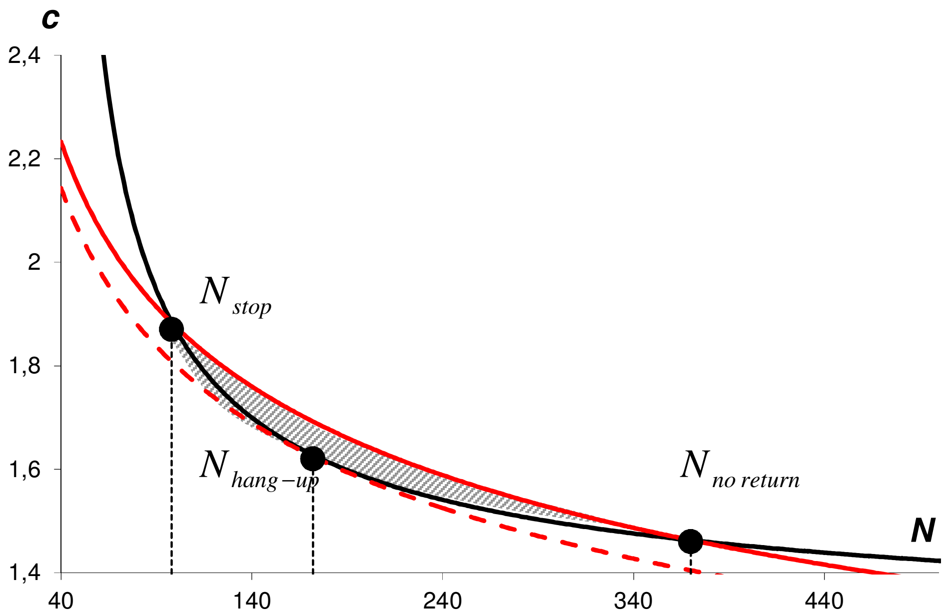

Figure 5: c as a function of N: $N_0 = 40$, $\alpha = 0.19, \gamma = 2.11$ and $\mu = 2.5$. Eq. 6 is represented by the black curve; Eq. 3 is illustrated both by the dotted red curve (for $c_o \sim c_{0,c}(\mu, N_0, \alpha)$) and by the red curve (for $c_o > c_{0,c}(\mu, N_0, \alpha)$).

Let us take the latter case, $c_o > c_{0,c}(\mu, N_0, \alpha)$. From Figure 5 Eq. 7 is expected to display initially negative values (Eq. 6 is above Eq. 2), then positive values from N_{stop} to $N_{no\ return}$ and finally negative values again. In other words, the process described begins at N_0 and it continuous undisturbed until N_{stop}. If not triggered the process would simply lie there and the technology would never conquer the entire market. Thus S_k has to be set such as to fill the gap between Eq. 2 and Eq. 6 for each N, which in this case, coincide with $k = 1,2,...,n$. Figure 6 below unveils the optimal dynamic path of subsidies S_k. The results support the argument that subsidies are needed at the outset of innovation. However, in the face of high-up front costs a new technology will never conquer the entire market if not triggered until the critical mass is reached, even if cost reductions are attainable in the future: subsidies have to last at least until $N_{no\ return}$ or any effort would be useless. The phasing-out stage is as important as the initial phase of the path: not only has the policy to sustain the process until $N_{no\ return}$ but it also has to give at least S_k at each iteration k. Any level of S_k lying below the curve in Figure 6 would be insufficient and the process would immediately converge again to the attractive fixed point N_{stop}.

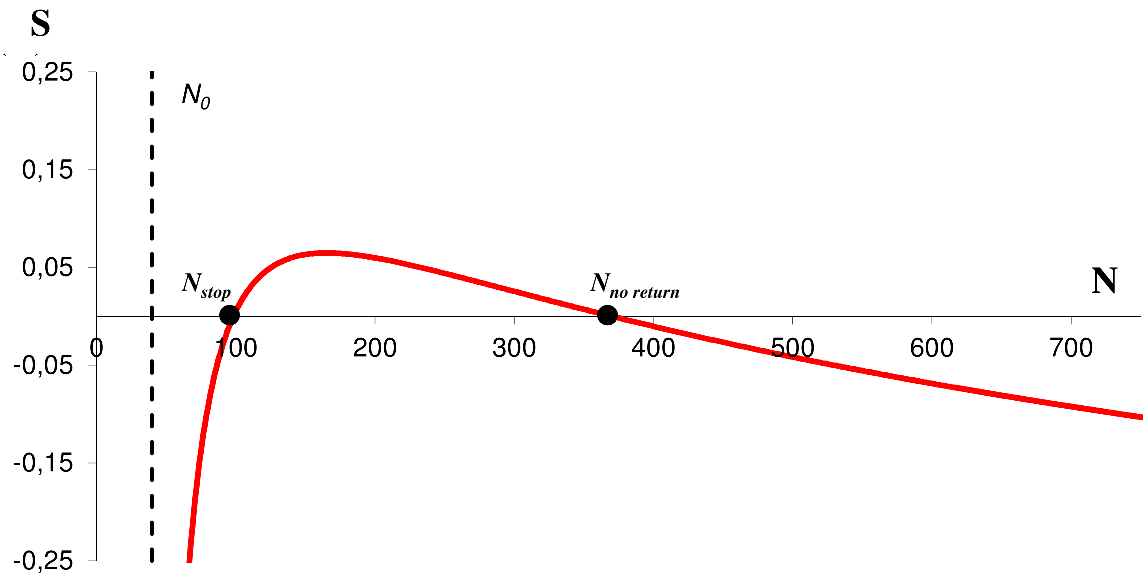

Figure 6: the dynamic optimal path of subsidies. $N_0 = 40$, $\alpha = 0.19, \gamma = 2.11$ **and** $\mu = 2.5$

In the next section the predictions of the model are compared with the Monte Carlo simulations in order to answer to the following questions. Do the main mechanisms of the model lead to the same conclusions and interpretations? What if Eq. 3 would not reproduce the peer-to-peer mechanism correctly? What about the reliability of the phase transition curve depicted by Figure 3? Finally is the optimal path of subsidy attainable?

V. THE SIMULATION MODEL

What looks like a smooth continuous function (Eq. 3) is infact a discrete process. The discrepancies between the former and the latter, though not so much statistically significant (see Cantono and Solomon, 2010), are actually responsible for the failure or the success of a technology, as well as for the effectiveness of a subsidy policy. Let us show how that works by describing the simulation version of the model.

M firms indexed by $i = 1, 2, 3, \ldots, M$ [10] are randomly spread at each node of a bi-dimensional regular lattice (i.e. *Ising* network) with periodic boundary conditions. Each firm i is connected to its business partners j. The links (i, j) forms the network geometry which characterizes the peer-

[10]In the model M is considered "large enough" for all the relevant ranges of the parameters, in particular $M = 10000$. In the present framework the effect of finite size M will not be discussed.

to-peer mechanism. To each firm i is assigned a gross benefit b_i drawn from a Pareto probability distribution:

$$P(b_i > c_k) = c_k^{-\mu} \text{ for } c_k > 1 \text{ and } P(b_i > c_k) = 1 \text{ for } c_k < 1$$

At each simulation time step k, each firm adopts if and only if 1) one of its business partner has already adopted the technology and 2) its gross benefit is higher than the adoption cost of the technology, $b_i > c_k$. As assumed in the mathematical model, adoption is a best response strategy if $b_i > c_k$ conditional on receiving a signal from at least a business partner who has already adopted. The cost of adoption dynamics follow Eq. 2.

Once the initial cost of adoption c_0 and the learning coefficient α are set, an initial number of adopters N_0 is chosen regardless their gross benefits and the simulation model runs according to two type of algorithms:

A. mesoscopic algorithm: at each simulation time step k, rules 1 and 2 follow one another until the process stops, then the cost of adoption is updated according to Eq. 2. The two procedures are repeated continuously until no more potential adopters join the diffusion cluster. Finally the data are recorded;
B. invasion percolation algorithm (Wilkinson and Willensen, 1983): firms neighbouring the percolation clusters are collected at each unitary increase in N. The one characterized by the highest gross benefit is selected and its status is switched to that of adopters even if $b_i < c_k$; the procedures are continuously implemented until $N = M$.

As Figure 7 illustrates, by running the invasion percolation algorithm a disorderly series of points appears (black points). Only a limited number of components are key element to the system dynamics (yellow points): those responsible for the destiny of the technology are the points (firms) whose b_i are lower than any of the previous $b_i s$ appeared in the noisy series, i.e. the lower envelope (Cantono and Solomon, 2010). These points seem to lie perfectly on the curve (red curve in the figure) which describes the required level of the cost of adoption c in order for the diffusion cluster to be of size N (Eq. 6 fitted to the given single configuration, i.e. $\gamma = 2.16$). Graphically the similarity between the cluster size dynamics and the lower envelope looks striking. But a careful glance reveals unevenly distributed holes along the curve. The holes are the isolated clusters of potential adopters met by the propagation process and connected by the firms on the lower envelope. The simulation model unveils cluster size dynamics. The sparse character of the lower envelope is the result of clusters fusion. Diffusion occurs along sudden explosions, irregular in strength, connected by a limited number of critical individuals. The reason why, given the set of parameters, the diffusion wave cannot instantaneously reach every potential adopters is thus the presence of non-mixing communities. Non-connected populations of potential adopters emerge out of the intertwining between the network of interactions and firms' heterogeneity. An example of the effect of isolated clusters of potential adopters is shown by the small quadrant in Figure 7. The yellow point on the left may obstruct the diffusion process: the island of firms which follow thereafter (black points in the quadrant) may not be able to consider the technology. If the cost of the new technology is not lower than its gross benefit by the time the wave of diffusion reaches it then the firm (represented by the yellow point in the quadrant) will not join the cluster of adopters and the process will stop.

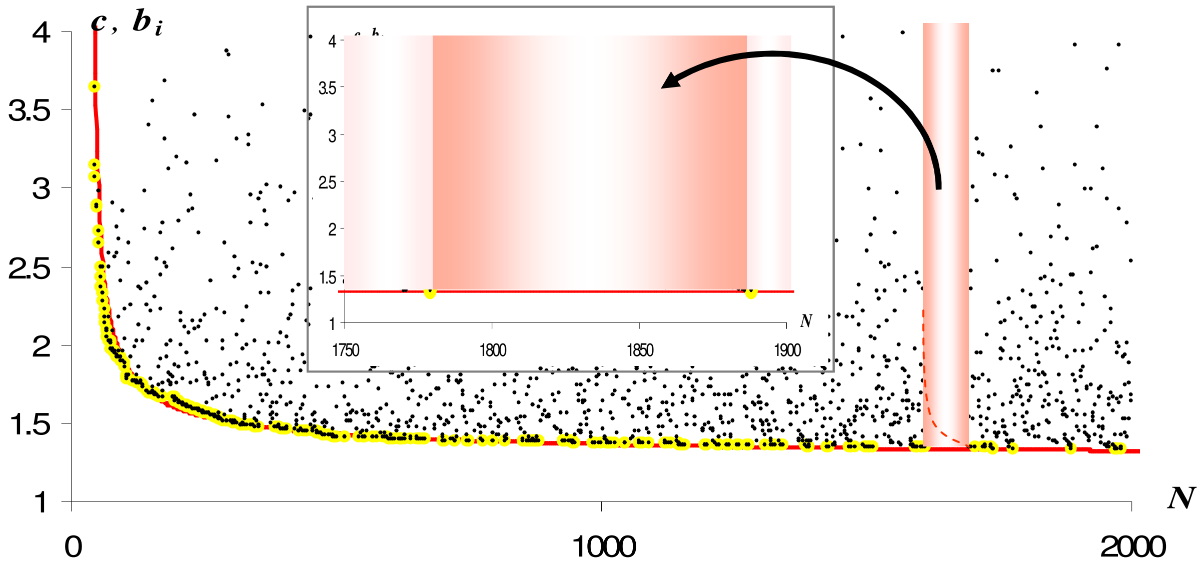

Figure 7: The lower envelope for a given configuration of gross benefits ($\mu = 5/2; \gamma = 2.16; N_0 = 40$)

Both the main mechanisms underlying the system dynamics and the predictions of the mathematical model have been verified by running the mesoscopic algorithm and i) there exists a discontinuous first order phase transition in $c_{0,c}$; ii) depending on the initial level of the cost the process may hang up at an apparently stable state of negligible diffusion; or it may propagate until overall diffusion is reached; or it may stop entrapped by isolated clusters of potential adopters. However the discrete character of the lower envelope is the source of the noise in the precision in of those estimations. Formally, for individual configurations, there will be situations in which the values of either $N_{hang-up}$, N_{stop} or $N_{no\ return}$ predicted by the mathematical model fall on one of the holes. In the simulation model the N corresponding to the closest previous point on the lower envelope will be recorded. Diverse efficient methods for reducing the noise have been explored and the predictions of the model have been validated (Cantono and Solomon, 2010). Yet it would be difficult to follow them blindly, for even if the noise can be reduced in a model it cannot be erased from reality.

Let us illustrate why this is so by taking one case of unsuccessful diffusion and by looking at the effect of the noise on the reliability of the optimal dynamic schedule of subsidies. Figure 8 shows the optimal path of subsidy according to Eq. 7 and the necessary minimum path of subsidy required for a given single configuration of gross benefits as it emerges from the simulation model.

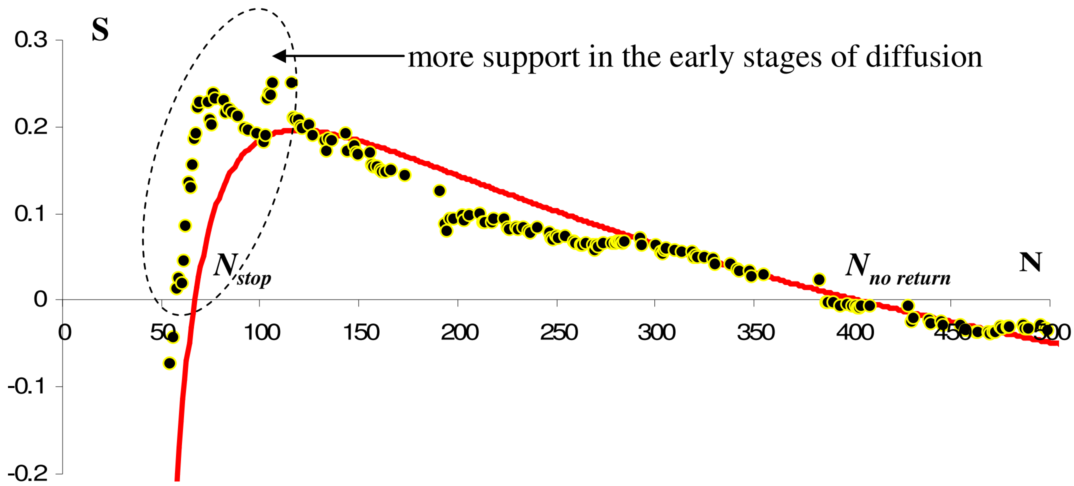

Figure 8: the red curve is the optimal dynamic subsidy schedule S as a function of N, according to Eq. 7; the yellow points represent the positive difference between the cost of adoption (Eq. 2) (vertical axis) and the level of the gross benefit for each corresponding N on the lower envelope (horizontal axis); the configuration of gross benefits is analogue to the one illustrated by Fig. 7; $N_0 = 40$, $\alpha = 0.25, \gamma = 2.16$, $\mu = 2.5$ and $c_0 > c_{0,c} = 6$

From Figure 8 it can be inferred that blindly following the dynamic path of subsidy advised by Eq. 7 will not help to support the diffusion of the new technology for the chosen system configuration. While the main mechanisms underlying the system dynamics are correctly reproduced and the main results holds (for $c_o > c_{0,c}(\mu, N_0, \alpha)$ the system stops either at a finite small N - N_{stop}, or it reaches overall propagation if forced until a critical N - $N_{no\ return}$; the necessary path of subsidies follows a similar path than the one predicted), the accuracy of the predictions is less comforting. Not only would the simulation model suggest to start the policy before (i.e. N_{stop} comes sooner than predicted) but it would also suggest a stronger intervention in the early stages of diffusion.

Arbitrarily setting the level of subsidy will likely trigger to a waste of resources. However, even if rigorously computed, any optimal dynamic schedule of subsidy will face the strength of uniqueness at any encounter with potential adopters' heterogeneity. In a limited number of cases, the specific characterization of potential adopters will be fatal. If not supported sufficiently, the wave of diffusion will neither rise nor fall on the coasts of the isolated clusters of potential adopters.

An overall synthesis of the main findings is given in the next and concluding section.

VI. CONCLUSIONS

To resume. Although boosted by future costs reductions, new technologies characterized by high up-front costs, such as many environmental technologies, may not be able to overcome the barriers to overall diffusion. Non-mixing communities may impede the free flow of information from past to future adopters often necessary for the adoption choice to take place.

The present model combines in a unique framework a learning curve model of dynamic cost reductions, a discrete choice model of heterogeneous technology adoption and a contagion model of technology diffusion. By focusing on the pattern of diffusion it uncovers its underlying dynamics. Isolated clusters of potential adopters emerge out of the interaction between heterogeneity and the network structure. New technologies which could potentially conquer the market may not be able to reach the ideal equilibrium because of limited communication within the system.

Hampered by such an obstacle, diffusion cannot be enhanced by potential future cost reductions. The intertwining between endogenous cost reductions, heterogeneity and the network of interactions through which information is transmitted gives rise to an autocatalytic feedback loop which renders the system dynamics path dependent and irreversible. Indeed there exists a discontinuous phase transition between localized and generalized propagation regimes. Under given conditions the propagation process may either be self-sustaining and conquer the entire market or it may stop at an insignificant diffusion level because entrapped by isolated communities of adopters. In the latter case the system ends up either at an apparently stable state, ready to be triggered to overall diffusion by the slightest disturbance, or facing a stagnation region defined by diverging frontiers, the lower attractive while the latter repulsive.

When held back by the attractive lower equilibrium only exogenous forces can spur the process to self-sustaining diffusion by accompanying it at least until the critical mass of adopters is reached. The present work defines the optimal dynamic schedule of adoption subsidy which insures overall propagation. The results support the argument that subsidies are needed at the outset of innovation however the phasing-out stage is as crucial. Subsidies should follow an initially increasing path and should decrease gently. Bu they should last at least until the repulsive upper equilibrium or any effort will be useless.

The reliability of the results from the mathematical model have been checked through the comparison with the Monte Carlo simulations. While on the one hand it emerges that the aggregate properties of the system (such as $c_{0,c}$ $N_{hang-up}$, N_{stop} and $N_{no\,return}$) are independent on the individual configuration, on the other hand only few firms are actually responsible for the fate of the technology (i.e. those on the lower envelope). The simulation version unveils the destabilizing forces acting at the individual level. Individuality is hardly reproducible, no matter how complex a model is: successful diffusion depends on a limited number of components and small changes in the individual configuration may be fatal.

The model presented in this article contributes to the literature by unveiling an emerging feature underlying diffusion dynamics, i.e. that of non-mixing populations (Karshenas and Stoneman, 1993; Foellmi and Zweimuller, 2006). Secondly, in the spirit of recent important contributions (Jackson and Yariv, 2007; Young, 2009), it focuses on the dynamic pattern of diffusion and it characterizes the aggregate equilibria of the system. Thirdly it offers the optimal dynamic schedule of subsidies which insures self-sustaining diffusion.

VII. APPENDIX

Eq. 5 can be rewritten as:

$$\left(\frac{c_c}{c_0}\right)^{\mu} N_0^{\alpha\mu}\left[\left(\frac{N}{N_0}\right)^{1/\gamma}\right]^{\alpha\mu\gamma+1} - \left(\frac{N}{N_0}\right)^{1/\gamma} + 1 = 0 \qquad [A.1]$$

Let us give the following definitions:

$$\left(\frac{c_c}{c_0}\right)^{\mu} N_0^{\alpha\mu} = \frac{1}{a} \qquad [A.2]$$

$$\left(\frac{N}{N_0}\right)^{1/\gamma} = x \qquad [A.3]$$

$$\alpha\mu\gamma = z \qquad [A.4]$$

By substituting A.2, A.3, A.4 into Eq. A.1 and rearranging the coefficients, we obtain:

$$x^{z+1} - ax + a = 0 \qquad [A.5]$$

Eq. A.5 can be easily solved for $z = 1$:

- there exists a critical stage which separates between regimes with real versus imaginary solutions, i.e. $a = 4$ and $x = 2$. It follows that $N_{hang-up} = 2^{\gamma} N_0$ and $c_{0,c} = 4^{1/\mu} c_c N_0^{\alpha}$; in other words, if the pace of learning curve cost reductions, the characterization of heterogeneity and the network structure (α, μ and γ respectively) are such that $\alpha\mu\gamma = 1$ and the initial cost of adoption is $c_0 = c_{0,c} = 4^{1/\mu} c_c N_0^{\alpha}$, the new technology does not diffuse and hangs up at an insignificant propagation level $N = N_{hang-up} = 2^{\gamma} N_0$;
- for $a >> 0$ the roots of Eq. A.5 are $x \sim 1$ and $x \sim a$. There exist two solutions the attractive $N_{stop} \sim N_0$ and the repulsive $N_{no\ return} \sim (c_c/c_0)^{1/\alpha}$;
- if the radicand is negative then the technology conquers the entire lattice.

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