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Making Amartya Sen's Capability Approach Operational:
A Random Scale Framework for Empirical Modeling

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by

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Abstract

Amartya Sen has developed the so-called capability approach to take account of the view that income on its own is not enough to measure economic inequality. This is because knowledge about people's income does not tell us what they are able to acquire with that income. For example, people with the same income may not have access to health and transportation services, schools and opportunities in the labor market.

Recently, there has been growing interest in empirical studies based on the capability approach. Most of these, however, are only loosely related to quantitative behavioral microeconomic theory, at least in a concrete and empirically operational way. The purpose of this paper is to demonstrate that the theory of random scale (utility) models (RSM) offers a suitable and powerful framework for representing and accounting for some key aspects of Sen's theory. In this paper we reinterpret the concepts Sen has proposed within the RSM framework, with particular reference to representations that are operational in empirical contexts.

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1. Introduction

The purpose of this paper is to consider aspects of a methodological approach to structural empirical analysis of applications based on Sen's capability theory. The capability theory proposed by Sen has been discussed in several papers and books: see Sen (1980, 1982, 1984, 1985a, 1985b, 1987, 1992, 1993, 1997) and Drèze and Sen (2002); see also Robeyns (2003) and Robeyns and Kuklys (2005). Recall that a central argument in Sen's capability approach is that economic inequality is not necessarily the same as income inequality: see, for example, Sen (1997). The reason for this is that knowledge about people's income on its own does not tell us about other things that matter for their welfare. People may be restricted in their choices as a result of discrimination, customs, moral codes, political regime, climate, infrastructure, transport, organization of health care, etc. For example, in many cities the risk of becoming a victim of violence restricts sports and social activities for women.

Recently, there has been a growing interest in empirical studies based on Sen's capability approach: see, for example, the references in Sen (1997), Robeyns and Kuklys (2005) and Anand et al. (2005, 2008). On the positive side, these studies focus on aspects that are often neglected in traditional economic analysis, such as the role of socio-economic environmental factors, the ability of individuals to take advantage of opportunities that are objectively available to them, and the relevance and various interpretations of the utility concept in the context of welfare analysis. However, these studies are typically only loosely related to quantitative microeconomic theory, at least in a concrete and empirically operational way. One obvious reason for this state of affairs may be that Sen himself is rather vague on matters regarding structural empirical strategies, and it is far from evident how such strategies should be devised. What further complicates matters is that Sen seems to reject utility theory as a sensible framework for welfare analysis, for the following reasons. First, he argues that individuals do not always rank choice alternatives in a manner perfectly consistent with that assumed by traditional utility theory. Second, the focus of traditional utility theory on conventional consumption goods is too narrow, since it neglects crucial qualitative aspects of choice and restrictions. Unfortunately, as Sen does not devise an alternative methodological strategy to replace utility theory, researchers who attempt to apply the capability approach in an empirical context are left pretty much on their own. This void obviously leads to ad hoc procedures, without much behavioral foundation. Of particular concern in this context is the link between choice alternatives and the corresponding choice set of the individual. As we know, in microeconomic theory this link is captured by the indirect utility representation (possibly constrained indirect utility).

In this paper we aim to demonstrate that the theory of random utility models, denoted by Random Scale Models (RSM), in fact offers a suitable and powerful framework for representing and accounting for some key aspects of Sen's theory. The reason we deviate from using the notion of random utility is because it may be viewed as requiring a utilitarian approach and may therefore be misleading. A random scale model can of course be interpreted as a random utility choice model. As an individual preference representation RSM are appealing because they do not require that agents are perfectly transitive. As is well known, this theory has turned out to be very useful in empirical analysis of choice behavior involving qualitative alternatives characterized by non-pecuniary attributes, and it can easily accommodate restrictions on choice opportunities in an operationally convenient way. But a utilitarian interpretation is not necessary. A random scale can, however, alternatively be interpreted as an "objective" scale, representing what is believed to be the objective value of the alternatives. With this interpretation, a random concept is still desirable, because the scale may vary over time and across suitable selected expert panels that are used to construct and evaluate the scale, in a way that is not perfectly predictable.

This paper is organized as follows. We start in section 2 by defining concepts and discuss how these are related to earlier work on capabilities. In section 3 we introduce the notion of scale randomness and clarify why this allows us to relax the assumption of transitivity and instead rely on the weaker assumption known as weak stochastic transitivity (transitivity on average). Section 4 reviews some implications of the RSM framework for probabilistic demand analysis (choice probabilities), such as how the probability of choosing a particular alternative depends on the choice set of available alternatives and their attributes, through suitable scale representation of alternatives. In sections 5 we show how the RSM framework can be used to characterize money-metric welfare measures as functions of the choice set, scale parameters that characterize preferences and depend on income, prices and other attributes that characterize the discrete alternatives. Section 6 considers simple examples and section 7 discusses an alternative RSM approach, originally proposed by van Praag (van Praag, 1968). Van Praag's approach is based on the dual preference or scale representation, which may in fact be interpreted as a random expenditure function representation.

2. Functionings, capability sets and choice constraints

Fundamental to Sen's development of the capability approach is his modification of choice and welfare theory by taking a closer look at some of the problematic aspects of the different meanings of utility concepts and choice opportunities and constraints. Assume now an RSM representation of "preferences". In a traditional utility approach the preferences are those of the individuals in the population under study, whereas in the alternative "objective" approach favored by Sen they are the preferences of experts chosen for establishing the scale.

A central issue in Sen's theory is the notion of capability and functioning. Traditional welfare analysis based on standard utility theory and demand analysis with focus on divisible quantities of goods, prices and income neglects important qualitative aspects related to discrete "qualitative" alternatives and to restrictions on the set of available qualitative alternatives. Examples of qualitative alternatives that are relevant in the context of welfare are different types of health and transportation service, schools, types of job in the labor market, residential location, types of housing, etc. The choice sets of such alternatives may vary a great deal across individuals due to qualifications, social networks, discrimination, environmental factors, geographical location, etc. It may also be the case that given agents are not capable of taking advantage of all the alternatives available to them. In other words, they may not be successful in utilizing the opportunities at their command. For example, disabled people may not be able to do many things able-bodied people can, even if they face the same choice set of alternatives. Furthermore, people who are illiterate will have no access to the content of many interesting books. Therefore Sen argues that any welfare measure should take into account the choice set of relevant and available qualitative alternatives, not think solely in terms of conventional consumer goods.

Bear in mind that the RSM approach can be adapted to a utility approach as well as to Sen's objective evaluation approach, where the latter is based on the evaluation of members of a chosen expert group. Suppose now that the individual agent (or alternatively the evaluation expert in Sen's approach) faces a countable set, C, of available choice alternatives where each alternative is characterized by a vector of characteristics (attributes): cf. Gorman (1956) and Lancaster (1966). The choice set C may depend on the environment and be agent-specific, and is supposed to account for the agent's ability, capability or will to take advantage of the opportunities offered to her or him. The choice alternatives may be very general, such as residential location sites, jobs, housing alternatives, schooling opportunities, marital partners, different lifestyles, commodities, transportation modes, etc. For example, in the labor market, the choice set of jobs may be agent-specific because education and qualifications determine to a great extent the choice set of jobs available to the person. In Sen's original terminology, the notion of capability (or capability set) is equivalent to the choice set *C*. In our approach, this will also be the definition of capability. The elements of the capability set will sometimes be called *functionings* and at other times simply alternatives or qualitative alternatives. This is consistent with Sen's definition of the concept of functioning.[2]

As Robeyns (2003) and Robeyns and Kuklys (2005) have pointed out, the notion of capability has been used in several different ways in the theoretical literature. For example, Nussbaum (1999)[3] has labeled potential functionings "capabilities". Robeyns (2003) also has noted that Sen in his later writings has used both our definition and Nussbaum's definition of capability.[4]

A major problem the researcher faces when trying to establish an empirical model is that a large number of characteristics and choice variables are unobservable. This limitation should be accounted for as much as possible and as early as possible in the formulation of theoretical concepts and quantitative representations. Note that an empirical model is also needed in the case of Sen's objective approach, because a sound methodology is necessary to accommodate the fact that different members of the selected expert group may rank functionings differently. We shall indicate how this can be done in specific cases.

3. Intra-individual randomness in preferences

Sen is uneasy about the concept of utility. Specifically, he does not believe that the standard assumptions of utility theory, such as completeness and transitivity, hold. Another related issue discussed by Sen is the problem of interpersonal comparisons. He also seems to be skeptical about the conventional subjectivist position of welfare assessment, as we have already mentioned. Sen (1985a, pp. 33–5) declares his belief that a purely subjectivist view of

[2] As Sen (1998, p. 15) writes: "A person's capability is, then, given by the set of alternative functioning vectors, from which the person can choose any one vector."

[3] Nussbaum also distinguishes between basic capabilities, internal capabilities, and combined capabilities. A basic capability set consists of the basic capabilities of human beings, such as the ability to reason, to imagine, to eat and speak, etc. An internal capability set represents restrictions on the set of capabilities that come from internal physical and psychological factors due to handicaps, deformed preferences and fears, such as the inability to break out of a violent relationship. A combined capability set represents restrictions due to both internal psychological factors and restrictions imposed by the family, social and religious conventions and the legal system.

[4] In his formalism, Sen (1985a, pp. 11–15) distinguishes between the commodity vector, the characteristics of the commodities and the utilization function (which is the different patterns of use of the characteristics associated with the commodities). This may be fine in a theoretical discussion for the sake of clarifying concepts, but it is questionable whether it is very helpful in the context of making the theory operational in practice.

well-being is “ultimately rejectable” and that “the limits of objectivity extend well into the assessment of well-being”. However, violations of transitivity may also be a problem within Sen’s objective approach. This is because there is no guarantee that different experts will share the same opinion about rank orderings of functionings. In fact, it will often be the case that different members of the expert group rank functionings differently.

In this section we shall focus on a probabilistic modification of the conventional approach. Obviously, probabilistic choice theory offers new opportunities to relax the rather strong consistency assumptions so central to conventional deterministic utility theory. As is well known, psychologists such as Thurstone (1927) proposed a random utility framework to deal with the observational fact that individuals often violate transitivity when faced with replications of (seemingly) identical choice experiments. His explanation was that individual decision-makers may be ambiguous about the precise utility of the respective alternatives, in the sense that if the same choice setting is repeated they may choose a different alternative. Thus the utility function can be viewed as containing a stochastic element that varies from one moment to the next in a manner that is not foreseeable to the individual agent. However, at each moment in time (each choice setting presented) the agent (individual or member of the expert group) will of course choose the alternative that maximizes momentary utility. Thus the so-called random utility – or discrete choice – theory developed by Thurstone (1927), Luce (1959), McFadden (1984, 2001) and many others is particularly designed to allow for this type of seemingly bounded rational behavior. Moreover, this theory has been proved to imply more practical methods for carrying out empirical analyses in many cases as compared to the conventional deterministic microeconomic theory. In particular, this is true for qualitative choice settings with a finite number of alternatives (discrete alternatives).

4. Choice models based on the random scale modeling framework

To fix ideas, let us now recapitulate some key concepts and developments within the theory of RSM to see how the theory of capability fits in and can be interpreted within this framework. Note that here the decision-maker may be either the individual agent, in the case where one departs from a utility approach, or a member of the expert group, in the case where one departs from Sen’s objective approach. Consider a countable set, *S*, of “universal” choice alternatives (functionings). By this we mean that *S* is the absolute maximal set of alternatives that are relevant, regardless of whether or not they are available to every agent in the population. Consider a particular agent (for simplicity we drop the indexation of the agent for now) with capability set *C*. As mentioned above, the capability set may possibly be individual-specific. For some individuals *C* may be equal to *S* (which means that they face no restrictions on their choice behavior), but for the overwhelming majority of individuals the capability sets will be subsets of *S*. We assume that the agent has preferences over the alternatives (functionings) in *S*. This assumption is crucial, for it means that even if the agent's capability set *C* is a proper subset of *S*, we may wish to conduct hypothetical counterfactual policy simulations in which the capability set is changed to a different choice set (different from *C*) within *S*. Let U_j denote the scale value of alternative *j* in *S*. Assume that this scale function has the representation $U_j = v_j + \varepsilon_j$, where v_j is an alternative-specific deterministic term, whereas $\varepsilon_j, j \in S$ is a random "error" term that is supposed to represent possible unpredictable variations in the agent's tastes across replications of identical choice experiments. Alternatively, the error terms can be interpreted as representing unobserved variations in tastes across agents. In fact, a more realistic scenario, consistent with the discussion in section 3, is that the random error terms may represent unobserved taste variations across agents as well as agent-specific temporal uncertainty in tastes.

Let us next consider the implications for the choice model under specific distributional assumptions of the random error terms. To make the exposition as simple as possible we assume here that the random error terms are i.i.d. across alternatives, with c.d.f., $exp(-exp(-x))$, (standardized type III extreme value distribution), for real *x*. As is well known, this distributional assumption implies choice behavior that satisfies the Independence from Irrelevant Alternatives assumption (IIA). This assumption is not essential for the arguments to follow and can easily be relaxed. Under this distributional assumption it follows from well-known results – for example, McFadden (1984) – that the probability of choosing alternative *j* from *C* can be expressed as

(1)
$$P_C(j) = P\{U_j = \max_{k \in C} U_k\} = \frac{\exp(v_j)}{\sum_{k \in C} \exp(v_k)}$$

for $j \in C$, and zero otherwise. This is the well-known Luce model – Luce (1959) – that McFadden developed into a practical econometric framework known as the Multinomial Logit model – see, for example, McFadden (1984). Bear in mind that both the choice set, which here is the capability set *C*, and the systematic terms $\{v_j\}$, representing the mean preferences, may be individual-specific. The term v_j will in empirical analysis often be specified as a function of observed characteristics of the agent and observed attributes of alternative *j*, such as alternative-specific prices and possibly other attributes characterizing the alternatives.

Once the systematic terms of the utility function (scale function) have been specified one can estimate the unknown parameters in the specification using well-known methods, such as the maximum likelihood procedure. The estimated model, represented by choice probabilities as in (1), can be applied for conducting policy simulation experiments to assess the effect on choice behavior by changing the choice/capability set *C*, or by changing some or all of the attributes in the systematic parts, $\{v_j\}$, of the scale function.

RSM can also be used to analyze data where one has information about preference rank orderings of alternatives. Recently, so-called Stated Preference (SP) surveys have become increasingly popular. In SP surveys individuals are asked to make choices, or alternatively to state the rank ordering of hypothetical alternatives presented. Now, suppose for notational simplicity that the choice set consists of four alternatives: that is, $C = \{1, 2, 3, 4\}$. Let $Q_C(h, j, k)$ be the probability that alternative *h* is the most preferred, alternative *j* is the second most preferred and alternative *k* is the third most preferred. Then we can show – see McFadden (1984) – that

$$(2)\qquad Q_C(h, j, k) = P\{U_h > U_j > U_k\} = \frac{\exp(v_h)}{\sum_{r\in C}\exp(v_r)}\cdot\frac{\exp(v_j)}{\sum_{r\in C\setminus\{h\}}\exp(v_r)}\cdot\frac{\exp(v_k)}{\sum_{r\in C\setminus\{h,j\}}\exp(v_r)}.$$

We notice that (2) can be expressed as

$$(3)\qquad Q_C(h, j, k) = P_C(h)P_{C\setminus\{h\}}(j)P_{C\setminus\{h,j\}}(k).$$

The interpretation of the right-hand side of (3) is quite intuitive. It tells us that the ranking of the alternatives in *C* can be interpreted as if the agent first chooses alternative *h* as the most preferred one from *C*, second chooses alternative *j* as the most preferred one from the remaining alternatives in $C\setminus\{h\}$ and finally chooses *k* as the most preferred one from the final remaining alternatives in $C\setminus\{h, j\}$.

Latent choice sets

The framework outlined above can readily be extended to the setting with latent choice sets, as developed by Ben-Akiva and Watanatada (1981), Dagsvik (1994) and Dagsvik et al. (2006). A good example is as follows. An agent faces the residential choice of which region to settle in. Let *S* be the total set of regions. Assume that not all the regions are available to a given agent. This may be relevant in the context of international migrations, where people from Third World countries have choice opportunities as regards migration that differ from those of people from developed countries. It is also relevant for migration within some countries, such as China for example, where there are restrictions on long-term residence in major cities. Let the regions be indexed by $j \in C.$ Given the choice of region, the agent must make other decisions related to job opportunities, schooling, recreational and child-care facilities, etc. We assume that these sub-alternatives within regions are not observable to the researcher and we call them elemental alternatives (Ben-Akiva and Watanatada, 1981). Let *k* = 1, 2, …, be an enumeration of all combinations of elemental alternatives, let K_j denote the corresponding choice set of latent elemental alternatives within region *j* and let m_j denote the number of elemental alternatives in K_j. In this example the combinations of regions and elemental alternatives are the functionings. It follows immediately from well-known results that the probability $P_C(j,k)$ that the agent will choose elemental alternative *k* in region *j* equals

$$P_C(j,k) = \frac{\exp(v_j)}{\sum_{r \in C} \sum_{q \in K_r} \exp(v_r)} = \frac{\exp(v_j)}{\sum_{r \in C} m_r \exp(v_r)} \tag{4}$$

for $k \in K_j$. From this it follows that

$$P_C(j) = \sum_{r \in K_j} P(j,k) = \frac{m_j \exp(v_j)}{\sum_{r \in C} m_r \exp(v_r)} = \frac{\exp(v_j + \log m_j)}{\sum_{r \in C} \exp(v_r + \log m_r)}, \tag{5}$$

where $P_C(j),$ as above, denotes the probability of choosing region *j* from *C*. We note that the choice probability in (5) of the observable choices has a form where the representative utility terms are weighted by the size of the respective choice set of elemental alternatives. Although the terms $\{m_j\}$ are unobservable, they can be represented by variables characterizing the size of the choice sets of elemental alternatives. How depends on the specific topic being studied.

5. Welfare measurement

In some of his writings Sen suggests that evaluation of well-being is a function of both the chosen functioning and the capability set (Sugden, 1993). Sen argues that in some instances capabilities may even be more relevant than functionings (Sen, 1992, p. 41). As discussed above, it follows from conventional deterministic utility theory that the value of the chosen alternative (indirect utility) will only depend on the choice set in cases where the highest-ranked alternative is not contained in the choice set. Otherwise it will be independent of the choice set. Consequently, following Sen, conventional theory is restrictive since it generally does not depend on the choice set apart from in special cases. The motivation for Sen's position is the casual observation, together with personal introspection, that even when alternatives that are ranked lower than the currently chosen one are removed from the choice set, the agent may still experience a psychological loss of "freedom". Unfortunately, Sen has very little to say about the crucial issue of how conventional theory should be extended to enable valuation of capability sets. However, the random scale approach taken in this paper offers a possible interpretation of how and why welfare evaluation should depend on the capability set and how a corresponding representation can be achieved.

Let us next proceed by discussing some crucial implications from random scale theory. The systematic term of the scale function, given alternative *j*, can often be expressed in the form

(6) $$v_j = g(y - w_j, z_j),$$

where y is income, w_j is the cost of alternative j, z_j is a vector of alternative *j*-specific attributes other than cost and g is a suitable function. As reviewed in section 3, we shall adopt Thurstone's setting by interpreting the random terms of the scale function as random to the agent her- or himself, in the sense that under repetition of seemingly identical choice experiments the agent may choose different alternatives on each occasion due to her or his difficulties with evaluating the precise value of the alternatives once and for all. Bear in mind, therefore, that although the agent maximizes the momentary scale function, the error terms may change over repeated choice settings because the agent may revise her or his previous utility evaluations in a manner that is not foreseeable.

Define the (constrained) welfare function as

(7) $$V_C(y,w,z) = \max_{r\in C} U_k = \max_{r\in C}(g(y-w_r, z_r)+\varepsilon_r),$$

where $w=(w_1,w_2,...,w_m)$ and $z=(z_1,z_2,...,z_m)$. Then it follows from well-known results that under the distributional assumption above the welfare function can be expressed as

(8) $$V_C(y,w,z) = \bar{V}_C(y,w,z)+\varepsilon_C(y),$$

where $\bar{V}_C(y,w,z) = EV_C(y,w,z) - 0.5772$ is the mean indirect scale and $\varepsilon_C(y)$ is a random term that is independent of $\bar{V}_C(y,w,z)$ and has the same distribution as the random terms $\{\varepsilon_j\}$, and

(9) $$\bar{V}_C(y,w,z) = \log\left(\sum_{r\in C}\exp(g(y-w_r,z_r))\right).$$

At first glance, one may be surprised to notice that the mean welfare function in the present setting always depends on the choice set, in contrast to the traditional theory. Bear in mind that in the traditional deterministic theory the welfare function will not depend on the choice set in cases where the choice set contains the highest-ranked alternative within *S*. The reason why the welfare function depends on the choice set in our approach stems from the property that the random scale function depends on stochastic terms. Intuition suggests that since the error terms may fluctuate randomly across replications of choice experiments, the average welfare function will depend on the choice set simply because in some instances the highest-ranked alternative in *S* may *not* be contained in *C*. Thus in some instances agents may not be constrained in their choice, whereas in other instances they may be constrained. As a result, the average welfare function will depend on the choice set. In contrast, if perfect transitivity were to hold, then the welfare function would not depend on *C* if the highest-ranked alternative in *S* belonged to *C*.

One possible explanation for such seemingly irrational perceptions may be the following. Since the agents' preferences are uncertain in the sense of Thurstone, they know that there is a chance that they may revise the scale evaluations of the alternatives several times in the future. Accordingly, the alternative that is chosen currently may not be the most preferred one at all future points of time, due to the influence of whims in perceptions and problems with assessing the precise value of the alternatives once and for all. Another alternative, other than the chosen one (and possibly one that may disappear), may thus be the most preferred one at some future point in time. The above argument is also applicable in the more general case in which the error terms are interpreted as representing intra- as well as inter-agent randomness (due to unobservable variables that affect tastes).

Another point worth mentioning here is the concern of practicality. Given that one interprets $\bar{V}_C(y,w,z)$ as the individual welfare function, we note that it has a very simple and convenient functional form, as a function of the choice set (capability set) *C*, income *y*, prices $\{w_k\}$ and other non-pecuniary attributes represented by $\{z_k\}$.

In the more general case with latent alternatives, the deterministic part of the welfare function in (9) takes the form

$$(10) \qquad \bar{V}_C(y,w,z) = \log\left(\sum_{r\in C} m_r \exp(g(y-w_r, z_r))\right).$$

The formalism above can be applied to define money-metric welfare measures, such as *Compensating Variation* and *Equivalent Variation*. To this end, suppose that all or some of the variables – income, capability set, prices and attributes – have changed from $y^0,\ C^0, \{p_j^0, z_j^0\}$ to $y^1,\ C^1, \{p_j^1, z_j^1\}$. Then one may define the corresponding Compensating variation (cv) measure, based on (9), by

$$(11) \qquad \bar{V}_{C^0}(y^0, w^0, z^0) = \bar{V}_{C^1}(y^1 - cv, w^1, z^1).$$

Thus the cv measure is the income compensation needed for maintaining the original scale level after the change in income, capability set, prices and attributes. This definition of cv is based on the mean indirect scale function. Morey et al. (1993) apply this approach. In a context where the random error terms are interpreted in the sense of Thurstone – that is, as due to individual uncertainty in scale evaluation of alternatives – it seems sensible to apply the definition in (11). However, if one interprets the random error terms as stemming to a large extent from unobserved extra-individual heterogeneity preferences, then it may be more appropriate to apply a money-metric concept that accommodates this type of heterogeneity. This is the approach taken by McFadden (1999), Herriges and Kling (1999) and Dagsvik and Karlström (2005). They define a random cv measure by means of the relation

$$(12) \qquad V_{C^0}(y^0, w^0, z^0) = V_{C^1}(y^1 - cv, w^1, z^1).$$

Whereas the cv measure based on (11) can be readily computed, it is much more difficult to compute the random cv measure based on (12). McFadden (1999) has developed a simulation procedure to this end. Dagsvik and Karlström (2005) have demonstrated that one can obtain analytic formulas for the distribution of cv based on (12).

6. Examples

To fix ideas and to illustrate the potential for applications, we shall discuss a couple of examples in this section. Although these are somewhat stylized they may be helpful for bringing out key concepts and building blocks of the approach.

Example 1

The purpose of this example is to illustrate use of the concepts of capability and functionings. We consider countries in which there may be insufficient basic schooling and possibly restrictions on the opportunities for women to marry without parental consent and to work outside the home, due to customs and religious codes. Let $S=\{f_1, f_2, f_3, f_4\},$ where the alternatives (functionings) are f_1 = (opportunities to marry according to own preferences and to work outside the home), f_2 = (opportunity to marry according to own preferences but not to work outside the home), f_3 = (no opportunity to marry without parental consent or to work outside the home) and f_4 = (no opportunity to marry without parental consent or to work outside the home). There may, for example, be a situation in which women face the following choice restrictions. Women in group 1, say, are not allowed to work outside the home but are able to marry without the consent of their parents. Women in group 2 are allowed to work outside the home and are able to marry without the consent of their parents. Women in group 3 are able to work outside the home but not to marry without the consent of their parents. Thus for women in group 1 the capability set is $C_1 = \{f_2, f_4\}$, for women in group 2 the capability set is $C_2 = S$ and for women in group 3 the capability set is $C_3 = \{f_3, f_4\}$.

In principle, one could imagine conducting a suitable SP survey for a sample of individuals or, alternatively, for members of an expert evaluation panel in order to illicit preferences over the functionings f_1 to f_4. Note that it is not evident that f_4 is the least preferred functioning by everyone. For example, in a country like Afghanistan, an expert panel consisting of orthodox Muslim members may assert that it is not good for women to marry against their parents' will and to work outside the home. Assume a random scale function is given, as in section 4. Then the probability of preferring function *j* is given by

(13) $$P_C(j) = \frac{\exp(v_j)}{\sum_{r \in C} \exp(v_r)},$$

where v_j j = 1, 2, 3, 4 is the mean scale of functioning *j*. The empirical counterpart of the choice probability $P_C(j)$ is the fraction of agents that prefer alternative *j* to *C*. By using the expression in (13) one can obtain maximum likelihood estimates of $\{v_j\}$. The parameters $\{v_j\}$ are unique up to an additive constant. Thus one can normalize such that, for example, $v_1 = 0$. It then follows immediately from (13) that the mean utilities can be expressed by the choice probabilities as

(14) $$v_j = \log\left(\frac{P_C(j)}{P_C(1)}\right).$$

The parameters $\{v_j\}$ define a unique scale that represents the respective average scales of the functionings in *C*. Note that this scale is not only ordinal but also cardinal in the sense that differences $v_j - v_k$, say, $j, k \in C$, are meaningful because it follows that

$$v_j - v_k = \log\left(\frac{P_C(j)}{P_C(k)}\right),$$

or, alternatively, the probability of preferring functioning *j* over *k* can be expressed as a function $v_j - v_k$, by

$$P_{\{j,k\}}(j) = P\{U_j > U_k\} = \frac{1}{1 + \exp(v_k - v_j)}.$$

Although the example above has been overly simplified and stylized, it shows how one can formulate and combine individual and aggregate preference representations. Furthermore, we have seen how these representations yield convenient and practical mathematical expressions for choice probabilities.

Example 2

In this example we focus on how to accommodate the impact of violence on women's behavior in urban areas after dark. Consider a particular woman and consider all possible social, entertainment, cultural and leisure activities. Without loss of generality we can list all suitable and possible combinations of activities. Let *S* be the index set of this list of combinations of activities. For simplicity we shall henceforth call the alternatives in *S* activities. In the terminology of Sen, the set *S* consists of all possible functionings. The woman faces a choice set *C*, say, of feasible combinations of activities that may be less than *S* because she may be reluctant to go out after dark. That is, the alternatives in $S \setminus C$ are viewed as not available by the woman due to the risk of becoming a victim of violence. Let U_k be the scale of activity *k* and assume for simplicity in this example that it has a very simple form, namely

(15) $$U_k = \alpha(y - w_k) + Z\beta_k + \varepsilon_k,$$

where *y* is income, w_k is the cost of activity *k*, *Z* includes age and other relevant individual characteristics and β_k is an unknown vector of parameters. The error terms $\{\varepsilon_k\}$ are all assumed independent with standard extreme value c.d.f., $\exp(-\exp(-x))$. The probability that the woman will choose activity *j* from *C* is therefore given by

(16) $$P_j = P(U_j = \max_{k\in C} U_k) = \frac{\exp(-\alpha(y - w_j) + Z\beta_j)}{\sum_{k\in C}\exp(-\alpha(y - w_k) + Z\beta_k)}.$$

For the sake of identification we need to normalize such that β_1, say, is equal to zero. Provided that we have a sample of individual observations on women's choices we can estimate the unknown parameters θ and $\{\beta_k\}$. From (9) it follows that the corresponding welfare measure is given by

(17) $$\bar{V}_C(y, w, z) = \alpha y + \log\left(\sum_{k\in C}\exp(Z\beta_k - \alpha w_k)\right).$$

By means of (11) and (17) one can assess the change in welfare of a reform that increases the actual choice set of activities. Specifically, the welfare loss compared to a situation in which the capability set is equal to *S* is, ceteris paribus, given by

$$(18) \qquad cv = \frac{1}{\alpha}\log\left(\sum_{k\in C}\exp(Z\beta_k - \alpha w_k)\right) - \frac{1}{\alpha}\log\left(\sum_{k\in S}\exp(Z\beta_k - \alpha w_k)\right).$$

7. Stated Preference data and the Leyden School approach

We have seen that the RSM approach we have outlined offers a powerful method for accommodating key aspects of Sen's capability approach. In the context of welfare measurement, our methodological approach assumes that data on agents' choices related to every relevant aspect of life are available. In practice, this may be difficult for several reasons. For example, people are usually constrained by the social, religious, climatic and political conditions of the country in which they live. In some countries climatic conditions produce heavy rainfall and flooding or periods of severe drought. Since many people cannot choose to go to another country or even another part of the country, it is hard to obtain revealed preference data on choices among, say, different climatic and political conditions.

An alternative to using revealed preference data is to conduct hypothetical interviews: the so-called Stated Preference method. The advantage of the SP method is that it enables the analyst to obtain information about agents' preference orderings by asking questions on preference rankings in hypothetical choice situations. By SP data one could, for example, obtain data on preferences in Example 1 over the functionings f_3 and f_4 for women in group 1 who do not have these opportunities in their capability set.[5]

In this section we shall discuss a method based on SP data that was originally proposed by van Praag and associates (Leyden School approach) and has recently been extended by Dagsvik et al. (2006). The original approach attributed to him was developed by van Praag (1968, 1971, 1991, 1994). Other contributions include van Herwaarden et al. (1977), van Herwaarden and Kapteyn (1981) and Kapteyn and Wansbeck (1985). A discussion and critique of their approach is given in Seidl (1994), to which van Praag and

[5] Some researchers express skepticism towards SP data because such data are not viewed as reliable as revealed choice data. It is also relevant that preferences may be experience-dependent, implying that decision-makers may find it hard to evaluate alternatives of which they have no experience.

Kapteyn responded in van Praag and Kapteyn (1994). We shall now give a brief summary of the approach of Dagsvik et al. (2006).

Assume an RSM and welfare function as discussed in sections 4 and 5. Let the expenditure function $Y_C(u)$ be defined by $V_C(Y_C(u)) = u.$ Note that since the scale function is random, so is the expenditure function. Evidently, since $V_C(Y_C(u)) = u,$ it follows from (8) due to the assumption of extreme value distributed random error terms that

(19) $$P(Y_C(u) > y) = P(V_C(Y_C(u)) > V_C(y))$$

$$= P(u > V_C(y)) = P(u > \max_{r \in C}(g(y - w_r, z_r) + \varepsilon_r)) = \exp(-\exp(\bar{V}_C(y,w,z) - u)),$$

with corresponding probability density function $f(y \mid u,w,z)$ given by

(20) $$f(y \mid u,w,z) = \left(\exp(-\exp(\bar{V}_C(y,w,z) - u))\exp(\bar{V}_C(y,w,z) - u)\right)\frac{\partial \bar{V}_C(y,w,z)}{\partial y},$$

conditional on the utility level, prices and other attributes.

Next let us describe a simple version of how the Leyden School proposes to collect the data. In their SP survey each respondent is asked to indicate (under his or her current conditions) what income level is needed for maintaining a specific utility level. The Leyden School typically uses six welfare levels, corresponding to “very bad”, “bad”, “insufficient”, “more than sufficient”, “good” and “excellent”. Thus $Y(u_j)$ is the expenditure necessary for achieving welfare level u_j , j = 1, 2, …, 6. This means that one can interpret the income levels the respondents report in the survey as realizations of the random variables $Y(u_j)$, j = 1, 2, …, 6 for each individual respondent. The welfare levels the respondents associate with the questions in the survey may be individual-specific, but in the empirical formulation we can with no loss of generality assume that the welfare levels u_j, j = 1, 2, …, 6 are equal across individuals and have the interpretation as mean population scale levels that correspond to the respective questions in the questionnaire: see Dagsvik et al. (2006). The reason for this is that we cannot separate individual differences in welfare levels from population heterogeneity in the mean welfare function.

In the theoretical development above we have only considered the indirect scale for a given income and the corresponding expenditure function for a given welfare level. Thus this theoretical framework cannot without further assumptions be applied to the type of SP data collected by the Leyden School approach. To this end Dagsvik et al. (2006) have developed an appropriate theory where the scale is viewed as a stochastic process of income: that is, a stochastic process with income as parameter. The corresponding dual representation is the expenditure function as a stochastic process with scale level as parameter. In this paper, however, we shall not discuss further details of the approach taken by Dagsvik et al. (2006). For expository simplicity, assume therefore in the following that we have answers that correspond only to one of the scale levels. With independent observations on the expenditure function, one could use the method of maximum likelihood based on the probability density in (20) to estimate the mean welfare function, $\bar{V}_C(y,w,z)$, provided that a suitable empirical specification of the function $g(\cdot)$ has been made. However, due to the fact that the Leyden School approach does not involve direct observations on the most preferred alternatives, identification of the welfare function may be more delicate and be sensitive to functional form assumptions.

We shall next consider a simplified specification of the welfare function. In Dagsvik et al. (2006) it is argued on the basis of behavioral axioms and some empirical evidence that the functional form of *g* in some instances should be a logarithm function, that is

(21) $$g(y-w_j,z_j)=\alpha\log(y-w_j)+h(z_j),$$

where α is a positive constant and $h(z_j)$ is a suitable function of the non-pecuniary attributes of alternative *j*. Assume now that the set of alternatives is very large, so that we can approximate the choice set with a suitable continuum. Assume furthermore that the costs belong to the set $[\mu_C,\infty]$ and that the non-pecuniary attributes belong to the set K_C. Assume there is a one-to-one mapping between the indexation of the alternatives and the values of the vector of non-pecuniary attributes. Let $m(z)$ be the counterpart of m_r in the continuous case. Hence we obtain from (21) that

(22) $$\sum_{r\in C} m_r \exp(g(y-w_r,z_r)) \cong \int_{w\in[\mu_C,y]}\int_{z\in K_C} m(z)\exp(h(z)+\alpha\log(y-w))dzdw$$

$$=\left(y-\mu_C\right)^{\alpha+1}H_C^{\alpha+1},$$

where

$$H_C = \left[\frac{1}{\alpha+1} \int_{z \in K_C} m(z) \exp(h(z)) dz \right]^{1/\alpha+1} .$$

From (9) and (22) it now follows that we can express the continuous counterpart of the deterministic term of the indirect utility as

(23) $$\bar{V}_C(y) = (\alpha+1)\log(y-\mu_C) + (\alpha+1)\log H_C$$

for $y > \mu_C$. Clearly, the term μ_C can be interpreted as a subsistence level. Thus we have demonstrated that our welfare measure can be expressed as income minus subsistence level μ_C, weighted with the function H_C which represents the value of non-pecuniary attributes. Both the subsistence level and the function H_C may depend on the choice set *C*. Consequently, it follows from (19) and (23) that

(24) $$P(Y_C(u) > y) = \exp(-(y-\mu_C)^{\alpha+1} e^u H_C^{\alpha+1}).$$

The distribution function given in (24) is in fact a Weibull distribution, with support above the threshold μ_C. The structure of the distribution in (24) implies that we can write

(25) $$Y_C(u) \overset{d}{=} \mu_C + \frac{\eta(u)^{1/(\alpha+1)} e^{u/(\alpha+1)}}{H_C},$$

where $\overset{d}{=}$ means equality in distribution and $\eta(u)$ is an exponentially distributed random variable with parameter equal to 1. This follows immediately by using (25) to calculate the c.d.f. of the expenditure function. By differentiating (24) with respect to *y* one obtains the corresponding probability density for the expenditure function, as with (20), and this density can be applied in a maximum likelihood estimation procedure to estimate the parameters α, μ_C and H_C. Note, however, that unless μ_C is determined a priori, this is a non-standard inference problem, because μ_C defines the boundary of the support. Now, suppose for a moment that we have data for different population groups and that within each group the level of μ_C is the same. As Flinn and Heckman (1982) have demonstrated, the maximum likelihood estimate of μ_C for a given group is the minimum of the observations within the given group. Once μ_C has been estimated it can be inserted into the likelihood function and one can estimate α and H_C in a second stage, given a specification of H_C.

If one is interested only in the aggregate welfare measure there is no need to estimate the parameter α. In fact, a simple estimation method is to apply OLS to estimate H_C, with $\log(Y(u)-\mu_C)$ as dependent variable, because (25) implies that

$$\log(Y(u)-\mu_C)=\tilde{u}-\log H_C+\omega_C(u), \tag{26}$$

where $\tilde{u}=(0.5772+u)/(\alpha+1)$, and the error term $\omega_C(u)$ is given by

$$\omega_C(u)=\frac{1}{\alpha+1}(\log\eta_C(u)-0.5772) \tag{27}$$

and has zero mean. Thus we have

$$E\log(Y(u)-\mu_C)=\tilde{u}-\log H_C. \tag{28}$$

Whereas H_C is individual-specific, $\tilde{u}$ can, as mentioned above, be interpreted as the population utility level (common to every person in the sample). With data for several utility levels (as in the Leyden School approach), one obtains estimates $\log H_C-\tilde{u}_j$, corresponding to welfare level $\tilde{u}_j$, where $\tilde{u}_j$ depends on welfare level *j* whereas H_C is individual-specific. Thus up to a suitable normalization one can identify and estimate $\tilde{u}_j$ for each *j* and H_C for each individual without further assumptions about H_C. In a second stage, the individual estimates of $\log H_C$ are used as dependent variables in a regression relation, with independent variables characterizing the individual's environment, including the individual's actual income (or the individual's income in the previous period). Specifically, a key point of the Leyden School approach is that it allows the researcher to assess the extent of how perceived well-being depends on current achieved income level. This is obtained by introducing, for example, current income as an independent variable in the regression relation mentioned above, through the specification of $\log H_C$.

So far we have assumed that the capability set *C* is fixed in the SP survey. But this is not necessary. In fact, by making repeated SP experiments for each person for every possible subset of the universal set *S*, one can in principle assess H_C, as a function of *C*, for *any* $C\in S$.

If the capability set contains many alternatives, however, this does not seem practical. But suppose now that the researcher has only a few selected policy simulation experiments (scenarios) in mind, regarding the welfare effect of changing capabilities. Suppose, moreover, that these counterfactual capabilities consist of a modification of the original ones (which may be unobserved by the researcher), in that selected alternatives are added to or removed from the capability set, corresponding to the respective scenarios of interest. One could then carry out separate SP surveys corresponding to each specific scenario and subsequently estimate the scenario-specific values of H_C.

It is clear that the Leyden School approach discussed above applies equally well to a setting where the sample of individuals is replaced by members of an expert evaluation panel and where each member of the panel is exposed to the SP questionnaire as explained above.

8. Welfare measurement based on life satisfaction data

Several researchers have conducted surveys in which individuals are interviewed about their overall life satisfaction on, for example, a seven-point scale: see Anand et al. (2005, 2008). Subsequently, data obtained in this way have been used to analyze the relationship between life satisfaction and selected covariates supposed to be relevant to life satisfaction. One version of this approach is, from a methodological point of view, similar to the Leyden School method. In this case, let $u_j, j = 1, 2, ..., 6,$ denote six welfare levels. Individuals in the survey are asked to report their satisfaction by welfare levels from 1 to 6. As above, let $V_C(y)$ denote the scale of an individual and let welfare level 1 correspond to $V_C(y) \le u_1$, welfare level j correspond to $V_C(y) \in (u_{j-1}, u_j],$ for j = 2, 3, …, 6, and welfare level 7 correspond to $V_C(y) > u_6.$ Whereas the scale levels $u_j, j = 1, 2, ..., 6,$ are to be interpreted as mean population welfare levels, the index $V_C(y)$ is an agent-specific random scale. If one assumes that $V_C(y) = \bar{V}_C(y) + \varepsilon_C(y),$ where ε is a random variable with c.d.f. F and $\bar{V}_C(y)$ is the corresponding deterministic part, we obtain that the probability that an agent will report satisfaction level j is equal to

(29) $$p_j = P(u_{j-1} < V_C(y) \le u_j) = F(u_j - \bar{V}_C(y)) - F(u_{j-1} - \bar{V}_C(y)).$$

Assuming next that eq. (8) and (23) hold, we then get from (29) that

(30) $$p_j = \exp(-e^{-u_j}(y-\mu_C)^{\alpha+1}H_C^{\alpha+1}) - \exp(-e^{-u_{j-1}}(y-\mu_C)^{\alpha+1}H_C^{\alpha+1}).$$

It can easily be verified that, subject to some normalization – for example, with $u_1 = 0$ – one can identify u_j, $j = 2,3,...,6$, α and H_C. The expression in (30) can be applied to estimate $\{u_j\}, \alpha$ and H_C, given a specification of H_C, provided that one has data on lifetime satisfaction and covariates that are suitable for characterizing H_C.

As with the Leyden School approach, the methodology outlined in this section can obviously also be used where data are obtained from expert panel evaluations.

9. Conclusion

The fundamental motivation behind Sen's capability approach – see, for example, Sen (1997) – is the fact that economic inequality is not necessarily the same as income inequality, because income may not always be informative about what people get from their income. In this paper we have addressed the challenge of formulating a structural decision-theoretic framework that is suitable for analyzing and assessing economic inequality based on Sen's capability theory and appropriate data. The point of departure is the probabilistic theory of discrete choice, in which the representation of preferences in terms of random scale functions is crucial. We have argued that the probabilistic feature of the theory of discrete choice allows for a representation of preferences of individuals or evaluation experts that avoids the strict assumption of transitivity. Furthermore, we have discussed how this theory allows for tractable representations of the *chosen* alternative (formulated as choice probabilities) that accommodates qualitative attributes of the choice alternatives (functionings) and choice sets (capabilities). Subsequently, we have discussed how one can establish tractable welfare measures that, in addition to being a function of income and prices, depend on the capability set, as well as on observable attributes (if such exist) of the functionings in the capability set. We have also discussed how a dual approach, based on the notion of random expenditure function and similar to the Leyden School approach, can be applied to obtain welfare measurements, provided that suitable SP data are available. Finally, we have considered how an RSM can be applied to estimate welfare representations in the case where one has data on life satisfaction.

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