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## BIASED TECHNOLOGICAL CHANGE: A CONTRIBUTION TO THE DEBATE

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# Biased Technological Change: A contribution to the debate

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## Abstract

Antonelli and Quatraro (2010) apply a specific methodology to identify the effects of biased technological change on productivity growth. However, this method has been criticized by Ji and Wang (2014). This research note is a reply to their critique.

**JEL classification:** O33

**Keywords:** Total factor productivity; Technological congruence.

## 1 Three compared models

This section describes three methods to measure total factor productivity (*TFP*) developed in Solow (1957), Antonelli and Quatraro (2010) and Ji and Wang (2014).

Solow (1957) assumes a generic aggregate production function of the form:

$$Y = F(K, L, t), \quad (1)$$

where  $Y$  is the output;  $K$  is the capital input in physical units;  $L$  is the labor input in physical units;  $t$  is the technical change i.e. “any kind of *shift* in the production function” (Solow, 1957:312).

Initially, Solow assumes a neutral technical change at time  $t$ , denoted by  $A_t$ . Formally:

$$Y_t = A_t \cdot f(K, L). \quad (2)$$

Nevertheless, although it does not represent the core of his paper,<sup>1</sup> Solow seemingly generalizes the function (2) in the case of a non-neutral shift of the production function, denoted by  $F_t$ . However, Solow makes a very restrictive hypothesis that brings back to a neutral shift of equation (1). More specifically, Solow assumes that:

$$Y_t = F_t \cdot f(K, L). \quad (3)$$

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<sup>1</sup>In fact, in Solow (1957) all the graphics, the econometric studies and the majority of the formulas derive from the hypothesis of a neutral shift.

He argues, correctly, that in order to change the function (1) to the function (3), it is necessary to assume that  $F_t$  is independent of  $K$  and  $L$ .<sup>2</sup>

In further calculations, Solow implicitly assumes a Cobb-Douglas production function. In this paper I set the same assumption. To be, as generic as possible, I focus on equation (3), in which the function  $F_t$  comprises it the independence hypothesis. Let define  $\alpha_t$  and  $\beta_t$  are, respectively, the output elasticity of capital and of labor at time  $t$ . However, Solow assumes constant returns to scale, as in the two models analyzed below. So,  $\alpha_t = 1 - \beta_t$ . Then, the Solow's *TFP* can be written as:

$$TFP_t^S = F_t = y_t - k_t^{\alpha_t} - l_t^{\beta_t}, \quad (4)$$

where lowercase letters refer to the logarithm of the corresponding uppercase letter variables (e.g.  $y_t = \ln(Y_t)$ ). The superscript  $S$  indicates that equation (4) is the *TFP* identified by Solow. It is clear that the *TFP* measures exactly the neutral shift effect of technological change.

Antonelli and Quatraro (2010), henceforth AQ (2010), criticized the Solow's method. In particular, they point out that equation (4) does not take into account all the possible effects of technological change. More specifically, they state that the Solow's methodology does not consider the introduction of biased technological change (*BTC*) as a form of technological change. In fact, only when the output's elasticity is kept constant, the difference between the historic output ( $y_t$ ) and the actual theoretical output ( $k_t^{\alpha_0} + l_t^{\beta_0}$ ) can measure the effects of the introduction of *BTC*. That is, according to AQ (2010), the correct *TFP* is:

$$\begin{aligned} TFP_t^{AQ} &= y_t - k_t^{\alpha_0} - l_t^{\beta_0} \\ &= y_t - k_t^{\alpha_t} - l_t^{\beta_t} + k_t^{\alpha_t} + l_t^{\beta_t} - k_t^{\alpha_0} - l_t^{\beta_0} \\ &= F_t + k_t^{\alpha_t} - k_t^{\alpha_0} + l_t^{\beta_t} - l_t^{\beta_0}, \end{aligned} \quad (5)$$

where the superscript  $AQ$  indicates that the equation (5) is the *TFP* identified by AQ (2010). The difference between (4) and (5) comes from a different definition of theoretical output. If the new output of elasticity of the most abundant factor is higher (lower) than the previous one then the  $TFP_t^{AQ}$  increases (decreases). Nevertheless, this does not

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<sup>2</sup>This aspect, however, is unclear in the text and it is a possible source of misinterpretation. Indeed, after calculating the case of a technological change neutral, Solow writes: "So far I have been assuming that technical change is neutral. But if we go back to (1) and carry out the same reasoning we arrive at something very like (2a) [it is the result with the neutral case], namely

$$\frac{\dot{q}}{q} = \frac{1}{F} \frac{\partial F}{\partial t} + w_k \frac{\dot{k}}{k}$$

It can be shown, by integrating a partial differential equation, that if  $\dot{F}/F$  is independent of  $K$  and  $L$  (actually under constant returns to scale only  $K/L$  matters) then (1) has the special form (1a) [for this paper (2)] and shifts in the production function are neutral". In fact, if the emphasis is only on the first part of the quote, Solow seems to claim that the new formula describes a generic case of a non-neutral technological change. Instead, if the attention is on the whole quote, it is clear that this formulation is correct only with the equation (3). Then, Solow only considers the case of a neutral shift of the production function.

derive from the shift effect *à la Solow*, but from the biased effect of technology. So, I can write the  $TFP_t^{AQ}$  as the sum of two effects: the shift effect ( $TFP_t^S$ ) and the biased effect ( $BTC_t^{AQ}$ ), that is the object of the AQ (2010) paper. Then:

$$TFP_t^{AQ} = TFP_t^S + BTC_t^{AQ}. \quad (6)$$

For this feature AQ (2010) call the  $TFP_t^{AQ}$  as the *total-TFP*.

Ji and Wang (2014), henceforth JW (2014), criticized AQ (2010) paper.<sup>3</sup> In particular, they argued that  $TFP_t^S$  accounts for non-neutral shifts of the production function.<sup>4</sup> They state that the correct *TFP* is:

$$\begin{aligned} TFP_t^{JW} &= y_t - k_0^{\alpha_t} - l_0^{\beta_t} \\ &= y_t - k_t^{\alpha_t} - l_t^{\beta_t} + k_t^{\alpha_t} + l_t^{\beta_t} - k_0^{\alpha_t} - l_0^{\beta_t} \\ &= F_t + k_t^{\alpha_t} - k_0^{\alpha_t} + l_t^{\beta_t} - l_0^{\beta_t}, \end{aligned} \quad (7)$$

where the superscript *JW* indicates that the equation (7) is the *TFP* identified by JW (2014).<sup>5</sup> The main difference between (4) and (7) is, once again, in the different definition of theoretical output. Similarly to AQ (2010), their proposal leads to two effects: the shift effect and the biased effect.<sup>6</sup> So, with the exception of superscripts, equation (6) holds.

However, the decomposition presented in (7) is in contrast to the rest of the paper. Indeed, if the first part of the paper shows that Solow's method is applicable also in a non-neutral technological contest, the second part of the paper seems to sustain the opposite thesis.

In fact, the equation  $TFP_t^{JW} = TFP_t^S + BTC_t^{JW}$  is consistent with the idea of AQ (2010).

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<sup>3</sup>This criticism also concerns at Antonelli (2006, 2012) and Antonelli and Quatraro (2013). In particular, JW (2014) reject the AQ (2010) assumption that Solow considers only neutral shifts of the production function.

<sup>4</sup>Probably Solow is unclear in the paper on this aspect but, as I show above, Solow assumes only neutral displacements of the production function. However, the Solow paper may lead the reader to consolidate their wrong interpretation. Below, there are some examples of the above statement. "The reader will note that I have already drifted into the habit of calling the curve of Chart 2  $\Delta A/A$  instead of the more general  $\Delta F/F$ . In fact a scatter of  $\Delta F/F$  against  $K/L$  (not shown) indicates no trace of a relationship. So I may state as a formal conclusion that over the period 1909-49, shifts in the aggregate production function netted out to be approximately neutral. Perhaps I should recall that I have defined neutrality to mean that the shifts were pure scale changes, leaving marginal rates of substitution unchanged at given capital/labor ratios" (Solow, 1957:316). "For comparison, Solomon Fabricant has estimated [...]. Not only he does the usual choice of weights for computing an aggregate resource-input involve something analogous to my assumption of competitive factor markets, but in addition [...] seem tacitly *assume* (a) that technical change is neutral [...]" (Solow, 1957:317).

<sup>5</sup>However, a similar method has already been used by Bernard and Jones (1996).

<sup>6</sup>In reality, JW (2014) do not write the  $TFP_t^{JW}$  with a logarithmic form so they formulation would not allow split the two effects as a sum.

## 2 Why the new method is wrong

Equations (5) and (7) show that the Solow's method does not allow to calculate the biased technological change. However, the two models use two different definitions of  $TFP$ , therefore there are two possible formulations about the  $BTC$ . In particular, AQ (2010) calculate the  $BTC$  as follows:

$$BTC_t^{AQ} = TFP_t^{AQ} - TFP_t^S. \quad (8)$$

When  $BTC_t^{AQ}$  in a region is above (below) zero, then the direction of the technological activity is right (wrong). This can be observed in equation (5). Assume that an input, e.g.  $k_t$ , has a relatively high value and that the corresponding elasticity of output, e.g.  $\alpha_t$ , increases (decreases). Then, under constant return to scale, the other output of elasticity, e.g.  $\beta_t$ , decreases (increases). So, the biased effect includes two opposite effects. Indeed, the first effect (increase of  $\alpha_t$ ) implies an increase of efficiency of  $k_t$  but the second effect (reduction of  $\beta_t$ ) implies the reduction in relative efficiency of  $l_t$ . However, by  $k_t > l_t$ , the first effect is larger. So, the biased effect amplifies (reduces) the total effect. Therefore, the technological change is consistent with the factors endowment. Of course, the reverse is true if the input has a relatively low value.

Using the same methodology, JW (2014) calculate the biased effect through:

$$BTC_t^{JW} = TFP_t^{JW} - TFP_t^S. \quad (9)$$

Also in equation (9) the critical value is zero. When  $BTC_t^{JW}$  in a region is above (below) zero, then the direction of the endowment factors is right (wrong). Then, the technology is appropriate whether the technology progress is in accordance with the factors endowment. This can be observed in equation (7). So, the biased effect amplifies (reduces) the total effect. Of course, if the elasticity of the output has a relatively low value, the opposite is true. Note that both wording and intuition of  $BTC_t^{JW}$  are symmetrical to the results in AQ (2010). The only difference between the equation (5) and the equation (7) is, as already mentioned above, in the explanation of the biased effect.

The  $TFP$  calculated with the last methodology is hard to justify. Indeed, calculating the change of production factors to analyze the change in technology is at least audacious. In particular, there are several reasons for which the inputs increase endogenously and only one of these reasons derives from technological change. Then, the production factors vary not only if the technology changes, but also if the relative cost of factors and/or the budget endowment is modified. So, it may happen that the technological activity is characterized by the right directionality but factor endowment is characterized by the opposite directionality (and vice versa).<sup>7</sup> Therefore, the methodology in JW (2014) measures the true biased effect only if the factor's budget and the costs are constant over time. These problems do not occur in the methodology developed in AQ (2010).

In addition, only in equation (8) it is possible to observe if there is directionality of technological change. In fact, as noted above, the change of the factors is endogenous in

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<sup>7</sup>To better understand this argument, to see also the example in the Appendix.

the model and so it is the effect and not the cause of technological change. Finally, even if the change of factors is exogenous, there is no reason to sustain that the increase of a production factor leads to a reduction of the other factor.

As a final remark, the JW (2014) method might be useful if the focus of the paper is not on technological change but on how the direction of the technology influences the variation of inputs. Indeed, JW (2014) show how the inputs vary, but they do not give information about the motivation for this change. In others words, JW (2014) paper does not explain whether this change comes from a decrease of prices or an increase of efficiency. On the contrary, using the idea underlying the paper of AQ (2010), JW (2014) method can to distinguish if the factors' endowment increases by technological reason or by other reasons.

### 3 Conclusion

This paper aims at clarifying the idea presented by the AQ (2010). I show that the criticism of JW (2014) probably comes from a misunderstanding of the Solow's paper and it is unjustified. In particular, I show that the AQ (2010) model calculates the correct biased effect and that the method of JW (2014) calculates the true biased effect only if there are not variations in factor costs and/or budget.

However, the criticism is useful for two reasons. On the one hand, it helps to lend support to the goodness of the paper of AQ (2010). In fact, I show that the model of JW (2014) also criticizes the Solow's paper for the lack of the biased effect in his analysis. However, I demonstrate that the  $BTC_t^{JW}$  does not calculate the direction of technological change. More precisely, it indicates neither the direction nor the technological change.

On the other hand, the JW (2014) implicitly show how the factors endowment changes within a region and if this variation is consistent with the technology in that region. This result is useful, even although it stems from using the methodology in AQ (2010), to know if the increase (decrease) of a factor of production is due to an increase (decrease) of the effectiveness factor or it is due to other aspects not related with technology.

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## References

- [1] Antonelli, C. (2006). Localized technological change and factor markets: Constraints and inducements to innovation. *Structural Change and Economic Dynamics*, 17(2), 224-247. doi:10.1016/j.strueco.2004.05.002.
- [2] Antonelli, C. (2012). Technological congruence and productivity growth, in Andersson, M., Johansson, B., Karlsson, C., Lööf, H., (eds.), *Innovation and Growth - From R&D strategies of innovating firms to economy-wide technological change*, Oxford University Press, Oxford, pp. 209 – 232. doi:10.1093/acprof:oso/9780199646685.001.0001.
- [3] Antonelli, C. and Quatraro, F. (2010). The Effects of Biased Technological Change on Total Factor Productivity: Empirical Evidence From a Sample of OECD Countries. *The Journal of Technology Transfer*, 35(4), 361-383. doi:10.1007/s10961-009-9134-2.
- [4] Antonelli, C. and Quatraro, F. (2013). Localized Technological Change and Efficiency Wages across European Regional Labour Markets. *Regional Studies*, 47(10), 1686-1700. doi:10.1080/00343404.2012.690068.
- [5] Bernard, A. B. and Jones, C. J. (1996). Comparing Apples to Oranges: Productivity Convergence and Measurement Across Industries and Countries. *The American Economic Review*, 86(5), 1216-1238. doi: 10.1257/aer.91.4.1168.
- [6] Ji, Y. and Wang, Y. (2014). Some comments on Antonelli and Quatraro's paper of measuring effect of biased technology on TFP. *The Journal of Technology Transfer*, 39(2), 276-280. doi:10.1007/s10961-013-9310-2.
- [7] Solow, R. M. (1957). Technical change and the aggregate production function. *The Review of Economics and Statistics*, 39(3), 312-320. doi: 10.2307/1926047.

## 5 Appendix - A numerical example

Let us consider a simple numerical example that makes extreme assumptions to clarify why the AQ (2010) method is better than the JW (2014) one. Let us assume that, at time 0, exists a region characterized by the following production function:

$$Y_0 = K_0^{0.25} \cdot L_0^{0.75}, \quad (10)$$

and by the cost function:

$$100 = 1K_0 + 5L_0. \quad (11)$$

Standard optimization implies that the firm will produce  $Y_0 = 17$ .

If at time 1 the technology does not change but the wage decreases, from 5 to 4, then the firm will be able to produce  $Y_1 = 20$ .

The *TFP* changes in the three cases:

$$TFP_1^S = \ln(20) - \ln(17) = 0, \quad (12)$$



$$TFP_1^{AQ} = \ln(20) - \ln(20) = 0, \quad (13)$$

$$TFP_1^{JW} = \ln(20) - \ln(17) = 1.6. \quad (14)$$

So, the *BTCs* are:

$$BTC_1^{AQ} = TFP_1^{AQ} - TFP_1^S = 0, \quad (15)$$

$$BTC_1^{JW} = TFP_1^{JW} - TFP_1^S = 1.6. \quad (16)$$

Equation (15) shows that the technology has not moved neither towards the more productive factor nor towards the less productive factor. Indeed, it is easily observe that there was no change of efficiency.

Equation (16) shows that the growth of capital is consistent with the technology of the region. However, it gives wrong information to understand the direction of technological change. Of course, this conclusion is true also in the case of a change in the price of capital and/or a budget endowment variation.

This example shows than only the method introduced by AQ (2010) can avoid the above problems.